

Analytical and Experimental Approaches to Shot Peening

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ABSTRACT

This paper is a first attempt on analytical approach to shot peening. Shot peening improves the surface engineering quality by eliminating the tool marks, such as machining, grinding, stamping and other surface defects. Most importantly, the improvements of shot peening are produced by combination of compressive stress and cold work. Compressive stresses are beneficial in increasing resistances to fatigue failures, while the cold work effects of shot peening treatments can increase the surface hardness. Although shot peening is extensively used in the industry, its academic understanding is very low. Shot peening has been considered as a black art and black engineering in the industry. The focus of this research is to layout foundation of the shot peening research in academic world. Only then, will the research propel more systematically rather than via the conventional industrial trial and error approach. In this research, analytical approach with experimental verification is presented.

KEYWORDS

Shot peening Residual stress Surface engineering Cold work

INTRODUCTION

Shot peening has been used for decades as a measure to overcome surface engineering problems in various industries. Its improvements are produced mainly by combinations of compressive residual stress and cold work. Compressive residual stresses are known to be beneficial in increasing resistances to fatigue failures and corrosion fatigue, while the cold work effects of shot peening treatments can increase the surface hardness of many materials (Dounde et al., 2015). It is believed to be the most economical and effective method of producing and making surface residual compressive stresses to increase the product life of treated metal parts. The increased strength of treated parts allows for lighter-weight parts that exhibit high wear and fatigue resistance.

The process can be defined as work hardening to the surface of components by propelling streams of spherical shots to the surface. The surface layer of material yields plastically to generate residual compressive stress. Among the practitioners, it has been known well that many parameters influence the efficiency of shot peening process. These are the peening coverage, saturation, shot material, shot size, speed, and peening time (Higounenc, 2005).

The topic of shot peening is chosen due to the reason that fundamental understanding on this subject is low. For very long time, this subject has been regarded as a "black engineering" which

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for example the perceived of benefit using larger size is real but explanation is lack. Likewise, the speed is also beneficial to increase the effectiveness but the reasons of those benefits were not disseminated by those who understand science well. In a way this paper is promoting to the academics for the science of shot peening, instead of keeping the "black art of shot peening" in the industry. This attempt is also on the line with the policy among the shot peeners³ from long time ago.

ANALYTICAL SOLUTION

The starting point for the construction of the shot peening model is the solution of the problem for a single impact of a ball on a half-space 4. The initial data for the models are:

- V speed of flying shot balls;
- R radius of shot balls;
- ρ density of the shot ball material;
- E elastic modulus;
- μ Poisson's ratio;
- $\sigma_u = G_u(\mathcal{E}_u)$ material hardening curve.

A ball flying at a speed V hits the surface, contact forces develop in the contact zone, and the kinetic energy of the ball begins to be transformed into the elastic energy of the ball/half space and plastic deformation of the material to be shot peened. After the velocity of the ball becomes zero, the following condition is achieved:

$$\frac{mV^2}{2} = k(A\pi + A\mathbf{II}) \tag{1}$$

where

 $A\pi$ – is half-space energy;

Аш – elastic energy of a ball;

k – Correction factor, taking into account the influence of the roughness of the hydrodynamic film formed at the moment of impact.

Equation (1) is the starting point for the formation of boundary conditions for the contact problem with an inhomogeneous half-space. In order to calculate the work $A\pi$, it is necessary to find the stress distribution in the half-space. To do this, we solve a system of differential equations describing the elastoplastic behavior of the material. Taking into account the axial symmetry, the set of equations is most conveniently written in the cylindrical coordinate system (r, θ , Z), where r – radius, Z – axis, directed along the axis of symmetry, θ – polar angle.

It is assumed that the mass forces are absent after collision, so the equations of equilibrium are as follows:

$$\frac{d\sigma_r}{dr} + \frac{d\tau_{rz}}{dZ} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$
(2a)

$$\frac{d\sigma_Z}{dZ} + \frac{d\tau_{zr}}{dr} + \frac{\tau_{zr}}{r} = 0$$
(2b)

In what follows, the following notations are used:

 σ_r , σ_Z – radial and axial stresses; \mathcal{E}_r , \mathcal{E}_Z – radial and axial deformations; u, ω – radial and axial movements; $\tau_{Zr} = \tau$ – tangential stresses; $\gamma_{Zr} = \gamma$ – shear strain; θ – bulk deformation.

For the convenience of solving the equation (2a, 2b), they are transformed into:

$$\frac{d(\sigma_r r)}{dr} + \frac{d(r\tau_{rz})}{dZ} + \sigma_{\theta} = 0;$$
(3a)

$$\frac{d(\sigma_Z r)}{dZ} + \frac{d(\tau_{Zr} r)}{dr} = 0$$
(3b)

This is done to make it easier to approximate the system being solved on the Z axis. Indeed, the system (3a, 3b) can be correctly solved only under the condition that:

$$\lim_{r \to 0} r\sigma_r = 0, \quad \lim_{r \to 0} r\sigma_Z = 0 \tag{4}$$

These limiting relations are also established when finite-difference formulas are realized. To solve the problem, the finite volume of the half-space in the form of a cylinder of radius *R* and height *H* are selected. On the upper base of the cylinder, the boundary conditions are given in the displacements (Figure 1), which are: a) within the contact area $u = u_0$, $\omega = \omega_0$, where (u_0, ω_0) is the introduction of an indenter into a half-space at specified distances from the Z axis; b) outside the contact area it is free, that is, at the corresponding nodes the system (equation 4) is solved.



Figure 1. Diagram of contact area

In addition, in connection with the lack of friction, the tangential stresses on the surface are zero $\tau = 0$. On the lateral face of the cylinder r = R and on the lower base Z = H the boundary conditions are given in the stresses, which are calculated from the formulas of the theory of elasticity, namely, on the lateral surface:

$$\sigma_r = \sigma_r^{\text{ynp}}(R, Z); \tau = \tau^{\text{ynp}}(R, Z)$$
(5a)

at the bottom:

$$\tau_r = \tau_r(r, H); \ \sigma_Z = \sigma_Z^{\text{ymp}}(r, H), \tag{5b}$$

where σ_r^{ynp} , σ_Z^{ynp} , τ^{ynp} are calculated from the formulas for solving the problem of indentation by a spherical indenter into an elastic half-space. In this case, the dimensions of the cylinder must be chosen so that the area of plastic deformation is inside the cylinder. On the axis of symmetry r = 0 as boundary values we take the following conditions: $u_r = 0$, since on the axis of symmetry the radial displacement is zero; $\tau_{zr} = 0$ similarly, by virtue of symmetry.

To describe the plasticity processes, we use the equations of the theory of small elastoplastic deformations and the method of elastic solutions, which, after some transformations, can be written in the form of Hooke's law:

$$\mathcal{E}_r = \frac{1}{E^*} [\sigma_r - V^* (\sigma_\theta + \sigma_r)]; \tag{6a}$$

$$\mathcal{E}_Z = \frac{1}{E^*} [\sigma_\theta - V^* (\sigma_r + \sigma_Z)]; \tag{6b}$$

$$\mathcal{E}_{\theta} = 0, \quad \gamma = \frac{1}{\sigma^*} \tau, \tag{6c}$$

where:

$$E^* = \frac{\frac{\sigma_u}{\varepsilon_u}}{1 + \frac{3 - 2V}{3E} \cdot \frac{\sigma_u}{\varepsilon_u}}; \quad \sigma^* = \frac{\sigma_u}{3\varepsilon_u};$$
(7a)

$$V^* = \left(\frac{1}{2} - \frac{1 - 2V}{3E} \cdot \frac{\sigma_u}{\varepsilon_u}\right) / \left(1 + \frac{1 - 2V}{3E} \cdot \frac{\sigma_u}{\varepsilon_u}\right),\tag{7b}$$

Here σ_u – stress intensity; \mathcal{E}_u – strain intensity.

Then the solution of the problem in the theory of plasticity is reduced into solving a problem of the contact theory of elasticity with variable elasticity parameters, determined by the formula described in Equations (6) and (7), and the relationship between the elasticity parameters:

$$\sigma^* = E^* / 2(1 + V^*) \tag{8}$$

The solution of contact tasks is organized as follows:

The initial introduction of the ball into the half-space is specified and, with the given boundary
conditions, the method of variable elasticity parameters determines the stresses and
deformations in the material; the energy of the deformed half-space is then determined from
the formulas:

$$A = \int_{0}^{H} 2\pi \int_{0}^{R} rW dr dz, \qquad (9)$$

where W – energy density.

$$W = W_0 + W_{\phi},\tag{10}$$

where W_0 – bulk deformation energy; and W_{ϕ} – forming energy;

$$W_{\rm o} = \frac{1 - 2V^*}{3E^*} (\sigma_r + \sigma_Z + \sigma_\theta)^2;$$
(11)

$$W_{\Phi} = \int_{0}^{\varepsilon} \sigma_{u} \, \mathcal{E}_{u} d\mathcal{E}_{u} \tag{12}$$

The verification of the fulfillment of the condition is by taking into account the assumption that the ball is perfectly rigid and $A_{III} = 0$. If the condition does not hold, then we continue to insert the ball into the half-space, until the condition is met.

The method of variable elasticity parameters is as follows. In the first approximation, solve the usual problem of the theory of elasticity, when the variable elasticity parameters are constant $E^* = E$, $V^* = V$ since the system becomes ordinary Hooke's law. At the first stage, we use the analytical formulas. From the obtained value, we find the deformations $\mathcal{E}_Z^{(1)}$, $\mathcal{E}_r^{(1)}$, $\mathcal{E}_{\theta}^{(1)}$ and the stresses $\sigma_Z^{(1)}$, $\sigma_r^{(1)}$, $\sigma_{\theta}^{(1)}$. According to the last values, at each point we determine the stress intensity $\sigma_u^{(1)}$ and the strain intensity $\mathcal{E}_u^{(1)}$. From the deformation curve we find the stress intensity $\sigma_u^{(1)^*}$, which corresponds to the calculated value $\mathcal{E}_u^{(1)}$, then we set $\sigma_1^* = \sigma_u^{(1)^*}/3\mathcal{E}_u^{(1)}$, and find the modules $E_1^* \bowtie V_1^*$, according to equations (7a, 7b).

In the second step we solve the problem of the theory of elasticity with the obtained elasticity parameters, we determine in the second approximation the displacement $u^{(2)}$ and $\omega^{(2)}$, then $\mathcal{E}_Z^{(2)}$, $\mathcal{E}_r^{(2)}$, $\mathcal{E}_{\theta}^{(2)}$ is $\sigma_Z^{(2)}$, $\sigma_r^{(2)}$, $\sigma_{\theta}^{(2)}$, from them we find the intensities $\sigma_u^{(2)}$, $\mathcal{E}_u^{(2)}$ at each point of space, we calculate $\sigma_u^{(2)*}$ by the deformation curve, we assume $\sigma_2^* = \sigma_u^{(2)*}/3\mathcal{E}_u^{(2)}$ etc.

The calculations are continued until the obtained results of the approximation calculations are different from the results (n - 1) of approximations by a given amount with the required accuracy. As the main criterion in the program, the condition, $|\sigma_n^* - \sigma_{n-1}^*| < \varepsilon$, which means virtually invariance of the elastic parameters, thus process converges.

The impact of a ball on a surface is a complex process, for the description of which it is necessary to use the equations of thermoelasticity, plasticity, impact theory, hydrodynamics. In addition, the presence of a complex surface profile, which is formed due to roughness, makes this task difficult to resolve. At the same time, as the analysis has shown, it is possible to identify the determining equations on which the behavior of the model depends, and discard the remaining nonessential bonds. Since the speed of flying ball in the process of hydrobasting is not high, then we neglect the dynamic effects, and assume that the shock is quasistatic. It is quasistatic if:

- the deformations are considered to be concentrated in the vicinity of the contact area and are determined by the static theory, the wave motion in the bodies is neglected;
- each body moves at any time with the velocity of its center of mass. The quasistatic conditions remain valid also in the case of plastic deformations, since the presence of plastic flow reduces the intensity of the contact pressure and, consequently, the energy going to the elastic wave motion. In the shot peening, the impact speed is known to be up to 70 m/s, it is possible to use the relations for inelastic contact stresses under static conditions with the yield stress is replaced by a dynamic yield strength. For this reason, we neglect the influence of thermal stresses, since even assuming that the entire energy of the ball is spent on heating, it still does not suffice to exert a significant influence on the distribution of residual stresses. In practice, we assume that the surface is perfectly smooth, and the expenditure of energy expended on the deformation of the scallops will be taken into account in formula (3a, 3b) by introducing corresponding corrections in the coefficient *k*.

Meanwhile, it is known from experiments and numerical calculations that the stress intensity is maximal on the axis of symmetry of the imprint and gradually decreases, tending to zero with increasing distance from the axis of symmetry of the print. With this in mind, for stress intensity, we can write the expression of:

$$\sigma_u = \beta(Z) \cdot e^{-\varepsilon r^2} \tag{13}$$

Applying similar arguments for residual stresses and taking into account that they essentially depend on the yield strength of the material, we obtain the expression:

$$\sigma_r^{\text{OCT}} = \theta_r(Z)(1 - \mu_r r^2)/k_r \sigma_T; \qquad (14a)$$

$$\sigma_{\theta}^{\text{OCT}} = \theta_{\theta}(Z)(1 - \mu_{\theta}r^2)/k_{\theta}\sigma_T, \qquad (14b)$$

where σ_T – yield strength;

 k_r and k_{θ} – coefficients of sensitivity of residual stresses to the yield strength of the material.

The functions $\beta(Z)$, $\theta_r(Z)$, $\theta_{\theta}(Z)$ are arbitrary, but based on physical meaning it is necessary to demand that:

Type of the valve determines the function of the τ againts time. In this study the function of the time τ is expressed in the following equation (Purohit et al., 2017; Gariépy et al., 2017):

$$\lim_{r \to \infty} \beta(Z) = 0; \tag{15a}$$

$$\lim_{r \to \infty} \theta_r(Z) = 0; \tag{15b}$$

$$\lim_{r \to \infty} \theta_{\theta}(Z) = 0 \tag{15c}$$

Figure 2 schematically shows the distribution of intensity of load and residual stresses in the area of prints, which are calculated from formulas (13) and (14). These formulas reflect the qualitative picture of the distribution of residual stresses.



Figure 2. Stress distribution in the reference area

We expand the function (13) into the Taylor series.

$$\sigma_{u} = \beta(Z) \left[1 + \sum (-1)^{2} \frac{1}{n!} \varepsilon^{n} r^{2n} \right]$$
(16)

If the yield stress is subject to the condition

$$\sigma_T \le \sigma_u^{max},\tag{17}$$

then in Equation (16) we can drop all terms except the first with a small error, as a result we obtain:

$$\sigma_u = \beta(Z)[1 - \varepsilon r^2] \tag{18}$$

Using the expression in Equation (18) we find the current radius of the hardening area:

$$\overline{\sigma_T} = \beta(Z)[1 - \mathcal{E}l^2],\tag{19}$$

Hence

$$l^{2} = \frac{1}{\varepsilon} \left(1 - \frac{\overline{\sigma_{T}}}{\beta(Z)} \right)$$
(20)

Now, taking into account equation (18), we obtain:

$$F(Z,l) = \int_{0}^{l} \alpha \beta(Z) r(1 - \varepsilon r^{2}) dr = \alpha \beta(Z) l^{2} \left(\frac{1}{2} - \frac{\varepsilon l^{2}}{4}\right),$$
(21)

While taking into account equation (19):

$$F(Z,l) = \alpha\beta(Z)\frac{1}{4\varepsilon}\left(1 - \frac{\overline{\sigma_T}}{\beta(Z)}\right)\left(1 + \frac{\overline{\sigma_T}}{\beta(Z)}\right)$$
(22)

Further, we take the integrals for F_r and F_{θ}

$$F_r = 2\pi \int_0^R r\theta_r(Z)(1-\mu_r r^2)/k_r \overline{\sigma_T}(Z)dr$$
(23)

Obviously, in order to take this integral, it is necessary to specify the limits of integration. The radius of integration R will be found from the expression:

$$\theta_r(Z)(1-\mu_r r^2)/k_r \overline{\sigma_T} = 0, \tag{24}$$

hence:

$$R = \sqrt{\frac{1}{\mu_r}}$$
(25)

Integrating (23) while taking into account (25), we obtain:

$$F_r(Z,l) = \frac{\theta_r(Z)}{4\mu_r k_r \overline{\sigma_T}(Z)}$$
(26)

Obviously, by carrying out similar actions for F_{θ} we obtain:

$$F_{\theta}(Z,l) = \frac{\theta_{\theta}(Z)}{4\mu_{\theta}k_{\theta}\overline{\sigma_{T}}(Z)}$$
(27)

We now substitute the obtained expression in equations (22), (26), (27) into equation (3).

$$\frac{d\overline{\sigma_T^{\Sigma}}(z,t)}{dt} = 2\pi q \left[\alpha \beta(Z) \frac{1}{4\varepsilon} \left(1 - \frac{\overline{\sigma_T^2}}{\beta^2(Z)} - \frac{1}{2\varepsilon} \left(1 - \frac{\overline{\sigma_T}}{\beta(Z)} \right) \right) \right];$$
(28a)

$$\frac{d\sigma_X^{\Sigma \text{ ocr}}}{dt} = \frac{2q}{\pi} \left(\frac{\theta_r(Z)}{4\mu_r k_r \overline{\sigma_T}} + \frac{\theta_\theta(Z)}{4\mu_\theta k_\theta \overline{\sigma_T}} \right)$$
(28b)

We now consider in more detail the first differential equation, since it does not depend on $\sigma_{(Z,t)}^{\Sigma \text{ ocr}}$, then it can be solved separately. For the convenience of the solution, we introduce the following coefficients:

$$k = 2\pi q; \quad a = \frac{k(r-\alpha)}{4\varepsilon\beta(Z)}; \quad b = \frac{k}{2\varepsilon}; \quad c = \frac{\alpha k\beta(Z)}{4\varepsilon}$$
(29)

Then the first equation (28) is transformed as follows:

$$\frac{d\bar{\sigma}_T(z,l)}{dt} = a\overline{\sigma_T^2}(z,t) - b\bar{\sigma}_T(z,t) + c$$
(30)

We shall seek a solution of (30) in the form of:

$$\bar{\sigma}_T = y_1 + \frac{1}{y'} \tag{31}$$

where y_1 – particular solution.

We assume that y_1 is a certain value that does not depend on the parameter t.

$$y_1 = Y \tag{32}$$

Then substituting equation (32) into equation (31) we obtain:

$$aY^2 + bY + c = 0 (33)$$

Solving this equation for Y we obtain two roots:

$$Y_{1,2} = \frac{\beta(Z)}{2-a} (1 \pm \sqrt{1 - a(2-a)})$$
(34)

Now (30) it can be reduced to a linear differential equation:

$$\frac{d\bar{y}}{dt} + (2aY - b)\bar{y} = -a \tag{35}$$

Substituting here (34) and taking into account (28), (29), we obtain the equation:

$$\frac{d\bar{y}}{dt} + \frac{k}{2\varepsilon}(A_0 - 1)\bar{y} + \frac{k(r-a)}{4\beta(Z)\varepsilon} = 0,$$
(36)

where

$$A_0 = 1 \pm \sqrt{1 - a(2 - a)} \tag{37}$$

For simplicity, suppose that the material is ideally hardened, then $\alpha = 1$ and (37) is transformed as follows:

$$\frac{d\bar{y}}{dt} + \frac{k}{4\beta(Z)\varepsilon} = 0 \tag{38}$$

Integrating (38), we obtain the expression:

$$\bar{y} = -(\frac{kt}{4\beta(Z)\varepsilon} + c_1) \tag{39}$$

Then (31) takes the following form:

$$\sigma_T = \beta(Z) - \frac{1}{\frac{kt}{4\beta(Z)\varepsilon} + c_1}$$
(40)

The constant c_1 is found from the boundary condition:

$$\bar{\sigma}_T|^{t=0} = \sigma_T^{\text{MCX}}; \tag{41a}$$

$$\sigma_T(Z,t) = \beta(Z) - \frac{1}{c_1}; \tag{41b}$$

hence c_1 is found:

$$c_1 = \frac{1}{-\sigma_T^{\text{HCX}} + \beta(Z)} \tag{42}$$

Substituting (42) into (41), we obtain:

$$\sigma_T = \beta(Z) - \frac{1}{\frac{kt}{4\beta(Z)\varepsilon} + \frac{1}{\beta(Z) - \sigma_T^{\mu_{CX}}}}$$
(43)

Study the solution obtained by us. If the hardening time increases, then: $\sigma_T(Z, t) \rightarrow \beta(Z)$ in the limit:

$$\lim_{t \to \infty} \sigma_T(Z, t) = \beta(Z) \tag{44}$$

Now consider the relationship between residual stresses and deflection of control plates Δf .

For simplicity, we take in (30) that $\mu_r = \mu_{\theta}$, $k_r = k_{\theta}$, then we obtain:

$$\frac{d\sigma_x^{\sum \text{oct}}(Z,t)}{dt} = \frac{2q}{\pi} \left(\frac{\theta_r(Z) - \theta_\theta(Z)}{4\mu k \sigma_r(Z,t)} \right)$$
(45)

We substitute into the equation (45):

$$d = \frac{2q}{\pi} \left(\frac{\theta_r(Z) - \theta_\theta(Z)}{4\mu k} \right),\tag{46}$$

then the equation takes the form:

$$\frac{d\sigma^{\sum \text{ocr}}(Z,t)}{dt} = \frac{d}{\sigma_T(Z,t)}$$
(47)

Equation (40) is reduced to the form:

$$\sigma_T = \frac{k\beta(Z)(\beta(Z) - \sigma_T^{\text{MCX}})t + 4\beta(Z)\varepsilon\sigma_T^{\text{MCX}}}{kt(\beta(Z) - \sigma_T^{\text{MCX}}) + 4\beta(Z)\varepsilon}$$
(48)

Taking into account (48), the equation (47) is becomes:

$$\frac{d\sigma_X^{\Sigma \text{ oct}}(Z,t)}{dt} = \frac{d[kt(\beta(Z) - \sigma_T^{\text{HCX}}) + 4\beta(Z)\mathcal{E}]}{kt(\beta(Z) - \sigma_T^{\text{HCX}}) + 4\beta(Z)\sigma_T^{\text{HCX}}\mathcal{E}}$$
(49)

We make the substitution in the equation (49):

$$a = k(\beta(Z) - \sigma_T^{\text{MCX}}); b = 4\beta(Z)\mathcal{E};$$
(50a)

$$f = k\beta(Z) \cdot (\beta(Z) - \sigma_T^{\text{HCX}}); g = 4\beta(Z)\sigma_T^{\text{HCX}}\mathcal{E};$$
(50b)

$$\frac{d\overline{\sigma_X^{\Sigma \text{ ocr}}}}{dt} = d\frac{at+b}{ft+g}$$
(50c)



Figure 3. Dependence of residual stresses on hardening time

We shall carry out a qualitative analysis of equation (50). At the initial time, the increment of residual stresses should be maximal, since there is no hardening, in the future it should decrease to a certain limit, which can be easily found from the relation in equation (50) (Figure 3).

$$\lim_{t \to \infty} \frac{d\sigma(Z, t)}{dt} = d\frac{a}{f}$$
(51)

The presence of this limit is easily explained by the existence of a maximum hardening limit:

$$\sigma_T < \sigma_T^{max}, \ 0 < t < \infty \tag{52}$$

This limit exists in many materials and is explained by its physical properties. We integrate now to equate to (50)

$$\overline{\sigma_X^{\Sigma \text{ ocr}}} = d \left[\frac{at}{f} + \frac{fb - ag}{f^2} \left(\ln(ft + g) - \ln g) \right]$$
(53)

The analysis shows that the calculated curve can be divided into 3 phases:

I. – fast initial growth phase;

II. – transitional period;

III. – saturation phase.

Since the solution of the adaptability equations at each i-th step of the integration requires the solution of the one-shot problem, it is most advantageous to apply the interactive solution methods, since the resolution of the equations at each new step begins with the previous solution, which increases the computational speed. In particular, here we have chosen a two-layer integration scheme for solving the problem. The main decision points listed below.

1) Based on the preset depth of ball penetration, determine the contact radius:

$$B = \sqrt{R_{\rm III} \cdot D_e},\tag{54}$$

where R_{III} – ball radius;

 D_e – depth of implementation.

- 2) Determine the number of nodes N1 in the contact area and check whether it falls into the required range, if not, the calculation is terminated. This is done to ensure that the number of grid nodes does not exceed the specified range, and, on the other hand, that the accuracy of the solution is sufficient.
- 3) Determine the coordinates of the grid nodes X (1), Y (1).
- 4) Determine the force of the ball pressing:

$$P = \frac{8\sigma_0 B D_e}{3(1-V)},\tag{55}$$

where σ_0 – initial shear modulus.

5) By the given force P and the radius of the print B determine the elastic stresses according to:

$$\sigma_r^{\text{ynp}} = \sigma_r^{\text{ynp}}(r, Z); \ \sigma_Z^{\text{ynp}} = \sigma_Z^{\text{ynp}}(r, Z); \ \sigma_\theta^{\text{ynp}} = \sigma_\theta^{\text{ynp}}(r, Z);$$
(56)

6) On the basis of (55) find the boundary conditions: on the axis r = 0 we obtain:

$$\sigma_r = \sigma_\theta = K \left[(1+V) \left(1 - Zarctg \frac{1}{Z} - \frac{1}{2(1+Z^2)} \right) \right];$$
(57a)

$$\sigma_Z = K \frac{1}{1+Z^2}; \ \tau = 0$$
 (57b)

On the surface Z = 0 we have $\tau = 0$

$$\sigma_r | r \le b = K \left[\sqrt{1 - r^2} + \frac{1 - 2V}{3r^2} \left(\left(1 - r^2 \right)^{3/2} - 1 \right) \right];$$
(58a)

$$\sigma_{\theta}|r \le b = K\left[\sqrt{1-r^2} + \frac{1-2V}{3r^2}\left(\left(1-r^2\right)^{3/2} - 1\right)\right];$$
(58b)

$$\sigma_Z | r \le b = K \sqrt{1 - r^2}; \tag{58a}$$

$$\sigma_{\theta}|r > b = -\sigma_r, \qquad \sigma_Z|r > b = 0 \tag{58b}$$

7) Find the boundary conditions on the lateral face:

$$F_{Gb}(b,Z) = \sigma_r^{\rm ynp}(b,Z); \tag{59a}$$

$$F_{\tau b}(b, Z) = \tau^{\mathrm{ynp}}(b, Z); \tag{59b}$$

$$F_{GN}(r,H) = \sigma_r^{\text{ynp}}(r,H);$$
(59c)

$$F_{\tau N}(r,H) = \tau^{\rm ynp}(r,H), \tag{59d}$$

where b, H – height and width of the calculation area.

- 8) On the basis of (58), find the elastic displacements at the nodes.
- 9) To solve the interatomic problem, take the calculated elastic stresses as initial conditions, in addition, assume at the boundary that the boundary conditions correspond to the conditions found from the elastic solution (58) and (59)
- 10) Determine the intensity of stresses taking into account the residual stresses $\overline{\sigma_X^{\text{OCT}}}$:

$$\sigma_{u} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_{r} + \overline{\sigma_{X}^{\text{OCT}}} - \sigma_{Z})^{2} + (\sigma_{\theta} + \overline{\sigma_{X}^{\text{OCT}}} - \sigma_{Z})^{2} + 6\tau^{2}}$$
(60)
$$\mathcal{O}_{U}$$
$$\mathcal{O}$$

Figure 4. Diagram of the method of variable elasticity parameters

To calculate the stress state in the plastic region, we use the variable elasticity method. According to this, the shear modulus σ and Poisson's ratio V at each point of the area are calculated (Figure 4). By σ^* and V^* their values depending on the value σ_u are indicated. σ and V are the values from the previous iteration. Find the intensity of the deformations by the formula:

$$\mathcal{E}_u = \frac{\sigma_u}{3} \left(\frac{1}{\sigma} + \frac{1 - 2V}{E} \right) \tag{61}$$

Further, we find the intensity along the deformation curve:

$$\sigma_u^* = a_n + b_n \mathcal{E}_u,\tag{62}$$

where n – number of the segment of the broken line, which approximates the curve, the deformation. After this, we determine the new value of the shear modulus by the formula.

$$\sigma^* = \frac{1}{\frac{3\mathcal{E}_u}{\sigma_u^u} + \frac{1 - 2V}{E}}$$
(63)

11) Find the deformations from the displacements, then calculate the stresses at the nodes from the obtained elastic deformations and the recalculated elastic modulus.

$$\sigma_r = \lambda \theta + 2\sigma \mathcal{E}_r; \tag{64a}$$

$$\sigma_{\theta} = \lambda \theta + 2\sigma \varepsilon_{\theta}; \tag{64b}$$

$$\sigma_Z = \lambda \theta + 2\sigma \mathcal{E}_Z \tag{64c}$$

12) To determine the displacements at the grid nodes, it is necessary to compile a system of linear equations with fixed elastic parameters:

$$AY = F,$$
 (65)

where Y – array of radial and axial displacements;

F – boundary conditions, which are determined from the equations (59)

a. In order to find the matrix A, it is necessary to express the equilibrium equations in the form of a displacement function. Substituting (64) into the equilibrium equations, we obtain a system of second-order differential equations:

$$\nabla^2 u + \frac{1}{1 - 2V} \cdot \frac{d}{dx} \left(\frac{du}{dx} + \frac{dV}{dy} + \frac{d\omega}{dz} \right) + \frac{F_x}{\mu} = 0;$$
(66a)

$$\nabla^2 V + \frac{1}{1 - 2V} \cdot \frac{d}{dy} \left(\frac{du}{dx} + \frac{dV}{dy} + \frac{d\omega}{dz} \right) + \frac{F_y}{\mu} = 0;$$
(66b)

$$\nabla^2 \omega + \frac{1}{1 - 2V} \cdot \frac{d}{dz} \left(\frac{du}{dx} + \frac{dV}{dy} + \frac{d\omega}{dz} \right) + \frac{F_z}{\mu} = 0$$
(66c)

The derivatives in (66) are found by three points:

$$y'(x_1) = \frac{1}{H}(-y_3 + 4y_2 - 3y_1);$$
(67a)

$$y'(x_i) = \frac{1}{H}(y_{i+1} - y_{i-1}), 1 < i < n;$$
(67b)

$$y'(x_n) = \frac{1}{H}(3y_n - 4y_{n-1} + y_{n-2}), \tag{67c}$$

here $y_i = y(x_i)$, H – double grid spacing.

Substituting the expression (67) into the equilibrium equation (66) and representing it as the product of matrices (65), we define the matrix A.

b. To solve the equation (65), we use the three-layer iterative method. According to this method, it is necessary to calculate the remainder by the formula:

$$r_k = AY_k - F \tag{68}$$

- c. Calculate the vector column. A_{rk} . For this we use the equations (65) and (66), but instead of displacement we substitute the remainder.
- Calculate the coefficients A, E, F and if K > 1 , then calculate the coefficients B, C,
 D:

$$A = (A_{rk}, r_k); B = (A_{rk}, r_{k-1});$$
(69a)

$$C = (r_k, r_k - r_{k-1}); D = (r_{k-1}, r_k - r_{k-1});$$
(69b)

$$E = (A_{rk}, A_{rk}); F = (r_k, r_k); F_{st} = (r_{k-1}, r_{k-1})$$
(69c)

e. Calculate the new vector of the solution Y at the -th integration by the the formulas:

$$Y^{(1)} = Y^{(0)} - \tau \cdot r_0, \, \text{при } k = 1 \tag{70a}$$

$$Y^{(k+1)} = Y^{(k-1)} + \alpha (Y^{(k)} - Y^{(k-1)}) - \alpha \tau r_k, \text{ при } k \ge 2,$$
(70b)

where

$$\alpha_k = \frac{(A-B)B - DE}{(C-D)E - (A-B)^2}; \ \alpha_1 = 1;$$
(71a)

$$\tau_1 = \frac{A}{E}; \ \tau_k = \frac{B}{E\alpha_k} + \frac{A-B}{E}, k \ge 2$$
(71b)

Since the contact task is being solved, it is necessary to fix the movement of the nodes in the contact area. In order for their values to remain unchanged in the iteration process, the discrepancy in these nodes is forcibly equated to zero, which leads to automatic fixation of displacements in these nodes.

The iterative process proceeds until the residual is sufficiently small, namely:

$$\left|\frac{F}{F_{st}} - 1\right| < \varepsilon, \tag{72a}$$

where \mathcal{E} – relative accuracy.

13) Calculate the new elastic modulus from formulas (61), (62), (63). Find the change in the elastic modulus at the (K + 1) iteration.

Further we check the condition: $\Delta E < \Delta E_g$, where ΔE – maximum increment of the shear modulus; ΔE_g – maximum increment error.

If the condition is not fulfilled, go to (10) and repeat the calculation again, otherwise go to the next step. This cycle limits the solution of the elastoplastic problem for a given depth of ball penetration.

14) Calculate the energy \exists of the half-space;

$$\vartheta = \int_{V} (\vartheta_{0} + \vartheta_{\phi}) dx dy dz; \qquad (73a)$$

$$\Theta_{\rm o} = \frac{1 - 2V^*}{6E^*} (\sigma_r + \sigma_\theta + \sigma_Z)^2; \tag{73b}$$

$$\Theta_{\phi} = \frac{\sigma_u \mathcal{E}_u}{2},$$
(73c)

where \exists_0 – bulk strain energy;

 $\boldsymbol{\vartheta}_{\boldsymbol{\varphi}}$ – forming energy.

- 15) Check whether the kinetic energy of the ball A_o is equal to the energy of the half-space. If not, go to step 17, otherwise go to the next step.
- 16) Define a new depth of the introduction of balls into a half-space, while:

$$h = h + \Delta h, \text{ if } \Im < A_o; \tag{74a}$$

$$h = h - \Delta h, \text{ if } \Im > A_o, \tag{74b}$$

where h – penetration depth of the ball;

 Δh – increment of penetration depth.

To achieve the convergence, the step is calculated using the following the formula:

If
$$\Im(h + \Delta h) > A_o$$
 and $\Im(h) < A_o$ then $\Delta h = \Delta h/2$ (75)

After recalculating the depth of implementation, go to step 1 of the algorithm. 17) Calculate the residual stresses by the formula:

$$\sigma_{ij}^{\text{OCT}} = \sigma_{ij}^{H} - \sigma_{ij}^{\text{ynp}},\tag{76}$$

where σ_{ij}^{H} – load stresses;

 σ_{ii}^{ynp} – stresses, discharges, obtained from the elastic solution.

The solutions obtained from the algorithm are based on the fact that at a sufficiently large distance from the contact zone the elastoplastic solution converges to the elastic solution. This assumption allows to limit the calculated area to a cylinder of the radius B and the height H, and at the nodes located on the outer boundary of the cylinder, the displacements obtained from the elastic solution.

Figure 5 shows the distribution of stresses with $K_{npon} = 1$, the ball radius $R_{\mu\nu} = 1,25 \ mm$. The figure shows the example of the residual stress distribution with a dummy mechanical property of the steel materials. For our experiment, the comparable stress would be the right bottom stress distribution. The overall computation can then be plotted alongside with experiment as shown in Figure 6.



Figure 5. Results of calculation of stresses for a single print



Figure 6. Results of analytical (dash line) and experimental (shown as error bars)

EXPERIMENT

The experiment was done with the appropriate shot ball size to mimic the analysis until saturation. The Almen strip measure was 0.55 mm. The coverage of the shot peening was approximately 98%. This is to mimic what is usually called saturated shot peening. Beyond this coverage, more shot peening amount that is put into the system will not affect the residual stress significantly. The samples were than profiled with an x-ray diffraction device. The measurement of residual stress with x-ray diffraction is based on measurements of changes in crystal lattice spacing, which manifest themselves as shifts in angular position of respective diffraction peaks, according to Bragg's law:

$$n \cdot \lambda = 2d \sin \theta \tag{77}$$

where

n is the reflection order

 λ the radiation wavelength

d the plane spacing, and

 θ is the diffraction angle.

The strain can then be computed by:

$$\varepsilon = \left(\frac{d - d_0}{d_0}\right) \tag{78}$$

where

 ε is the strain in a particular direction

d the stressed, and d_0 the unstressed interplanar spacing

Since the stress in this case is only a conversion of the strain and the strain is only obtained using the full width half max approach, what we obtained was only a value with error bar. In our case, we measurement was performed to obtain the stress in the surface direction. Figure 6 shows the results. The compressive residual stress produces more than 500 MPa at the surface, while the peak depth of compressive residual stress was approximately at 0.19 mm. This trend is similar with the analytical results as well as the finding of other researchers (Gariépy et al., 2017).

DISCUSSIONS

The analysis and the experiment were in a quite good agreement. There is some minor disagreement at certain area such as the depth of the crossing line where the residual stress change from compressive to tension. However, since this is our first attempt to academically materialize the concept of the shot peening in calculation such discrepancy is highly anticipated. Other researchers also pursue similarly (Higounenc, 2005; Chang et al., 2008; Hu et al., 2017; Bianchetti et al., 2019).

Among the practitioners, the saturated residual stress at the surface is usually predicted as:

Surface
$$\sigma_{RS}[\text{MPa}] = -276 \frac{Ah}{R} + 7.1 \gamma_R - 0.59 \sigma_{pre} - 451$$
 (79)

At the surface, where *Ah* is the arc height and *R* is the radius of shot, while γ_R is the retain austenite. For the peak, usually it becomes

Max.
$$\sigma_{RS}[MPa] = -172 \frac{Ah}{R} + 7.1 \gamma_R - 0.54 \sigma_{pre} - 882$$
 (80)

the unit is in MPa. The above empirical equations are quite famous in the industrial world although it is lack of scientific basis. Based on equations (79) and (80), and our measurement of Almen arc-height, our prediction of surface residual stress would be -530MPa. The peak residual stress would be -917MPa. Again, direct comparison of the prediction using these empirical equations with experiment and with computational results are still difficult. We can only say that the trend is similar. However, since this is a first attempt in bring the "black art" into academic world, such difficulty was expected. The most important thing in here is the analytical solution is now derived. This solution is off course far from perfect. It has a lot of room for improvement.

CONCLUSIONS

Through analysis and experiment, it is proved that shot peening can be brought into academic domain instead of staying in industry as a black art. This research proved that academically this field has so much promise and potential to develop further. Simple analytical approach using classical solid mechanics concept leads to the prediction of the residual stress that is verifiable through experiment. Using the data from the experiment as a value to achieve, the analysis still has a room for improvement.

The process can be defined as work hardening to the surface of components by propelling streams of spherical shots to the surface. The surface layer of material yields plastically to generate residual compressive stress. Among the practitioners, it has been known well that many parameters influence the efficiency of shot peening process. These are the peening coverage, saturation, shot material, shot size, speed, and peening time.

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