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Edge Crack in Longitudinal Butt-Welded Joint in Thick-Wall Cylinder

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ABSTRACT
Thick-wall vessels and pipes cylindrical shape are very typical in power plant, chemical, processing, oil and gas industry. The equipment with cylindrical shape can be either thin or thick wall which depends on the function of that particular equipment. Typically, thick-wall cylinder is used when the equipment is needed to accommodate high pressure contents. Mostly, cracks appear either on the internal or external of a thick-wall cylinder. Primarily, when welding is applied in the fabrication of the thick-wall cylinder, cracks can easily appear due to solidification or hydrogen embrittlement at the welded joint, typically butt-welded joint. Hence, it is critical to examine the stress distribution along the crack and resolve the stress intensity factor of the cracks in both welded and non-welded internally pressurized thick-wall cylinder. Finite element analysis has been conducted using the engineering software, ABAQUS CAE to investigate the stress distribution and to perform the evaluation of stress intensity factor. Besides, weight function method has also been used by other researchers to determine the factor of stress intensity for both welded and non-welded thick-wall cylinder. The results were compared in terms of both of the methods applied. The last, the effect of the butt-welded joint profile in thick-wall cylinder has also been investigated.

KEYWORDS
Factor of stress intensity
Weight function
Finite element analysis
Thick-wall cylinder
Crack
Butt-welded joint
Edge crack

INTRODUCTION

Thick-wall vessels and pipes with cylindrical shape are very typical in production, transportation or to process products. The reliability of the material and structure of the vessels are very important since they have to be ensured that the safety usage of these vessels do not jeopardize the health and safety of the people around them.

There are many pipes and pressure vessels in every industry, each of them are manufactured according to standard and design codes to meet the operation conditions. These design codes such as ASME Boilers and Pressure Vessels Section VIII (ASME, 2015) and ASME B31 Process Piping (ASME, 2006) are the two major standards to govern the standards of the

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equipment used in industry which consists of every study in terms of their material properties, design parameters, fracture mechanics, stress analysis and etc.

The study of stresses in the cylinder has always been very important because of the stress components may cause failure to the cylinder especially thick-wall cylinder since they are designed to contain high pressure fluids. Thick-wall cylinder theory is based on theory of elasticity which has been developed for a very a long time. This theory has been used for stress component calculation in the cylinder with thick-wall and was contributed by using the famous Lame's Theorem (Dr. B. C. et al., 2012).

Despite being abundant, studies of thick-wall cylinder in terms of stress analysis are not fully covered. Thus, there are studies focuses on the fracture mechanics in thick-wall cylinder. This direction is needed because it is important to understand how thick-wall cylinders fracture in service condition or even before service due to inspection and testing. There are many research studies have been conducted to study the fracture mechanics in thick-wall cylinder for a while (Underwood, 1972). These research findings focus on the crack type on the thick-wall cylinder and how they affect the field and intensity factor of stress at the crack region of the intensity factor with the thick-wall cylinder.

Crack usually is the main reason to cause fracture in thick-wall cylinder. Crack happens when there are micro-defects inside or outside the cylinder surface. Since imperfection at the cylinder surface always occurs, such micro-defects always grow to cause crack and thus failure to the cylinder. That is why there is a design method which is leak-before-break design approaches which to design the cylinder to have higher fracture toughness. One good example shown in Figure 1 is the pressure vessel that failed under hydro test. Such case should not have been happen as long it is manufacture to the design code strictly.

Figure 1. Crack occurs in a pressure vessel during hydro test (Source: www.wermac.org)

The more critical part is the welded joints at the thick-wall cylinder which affect the quality of the cylinder. Even with strict and high weld quality, assessment and inspection checks are done to ensure the welding procedures are done in correct way, there are still a chance of failure occur at the welded joints. The welded joints regardless of what type of weld, the material properties is changed around the welded region compared to the parent or base material. In addition, the weld joint shape profile also affects the stress distribution around the welded joint. This effect contributes stress concentration areas which are stress raisers, thus inducing cracks in the region.

Even the design of the equipment can withstand the design stress but the design method is based on the parent material. This means that even when the thick-wall cylinder can withstand the design stress, but the welded joints which have different allowable stress due to the different of material properties and the weld joint shape, failure may still occur at the welded joint when there is a crack exist in that region. One typical example is solidification crack which happens in butt-welded joint as shown in Figure 2.

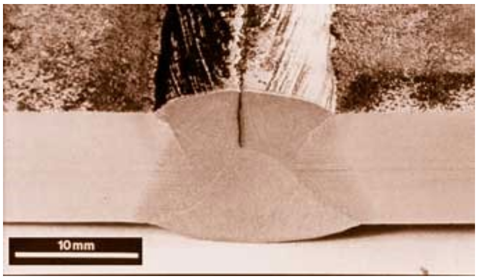

Figure 2. Solidification crack along the centre line of the weld (www.twi.co.uk)

Therefore in this research, it is about performing the static stress analysis of a surface crack on thick-wall cylinder and analyse the crack at the welded and non-welded crack on the thick-wall cylinder.

LITERATURE REVIEW

Stress Intensity Approach

Stress intensity factor refers to Linear Elastic Fracture Mechanics (LEFM) approach, the stress intensity is determined by the elastic stress field equations for a stressed element near the tip of a sharp crack under biaxial or uniaxial loading in an infinite body. Considering a crack tip area in a Mode I which is the opening crack propagation mode, taking an element at the crack tip and the stress field is shown by using complex stress function (Westergaard H.M., 1939), the opening-mode stresses are:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \cdots \tag{1}$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \cdots \tag{2}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\sin\frac{\theta}{2}\cdots \tag{3}$$

The K_I in the Equation 1 to Equation 3 indicated as the factor of stress intensity for Mode I crack mode. When come closer to the crack tip, $r \to 0$, the stresses go to infinity. In this case, it is called the stress singularity at the crack tip due to the denominator factor $(2\pi r)$-1/2. In general terms, since θ = 0 at the crack tip, thus the factor of stress intensity for Mode I shown as follow:

$$K_I = \sigma\sqrt{\pi a} \tag{5}$$

The variable a in the Equation 5 is the length of the crack on the part. Since the factor of stress intensity is a function of geometry, size and shape of the crack, and the loadings, there is a stress intensity modification factor, β that is introduced in the Equation 6:

$$K_I = \beta\sigma\sqrt{\pi a} \tag{6}$$

There are many research and findings of the value of β long ago thanks to the contribution of researchers and engineers. Table 1 shows some of the basic configurations (Tada et al., 2000) for stress intensity factor, KI.

Table 1. The factors of stress intensity for different crack configuration types

Crack Type	Factor of Stress Intensity, K_I
Centre crack with length $2a$ in an infinite plate.	$K_I = \sigma\sqrt{\pi a}$
Edge crack with length$a$ in a semi-infinite plate.	$K_I = 1.12\sigma\sqrt{\pi a}$
Central penny-shaped crack, radius a, in an infinite body	$K_I = 2\sigma\sqrt{\frac{a}{\pi}}$
Centre crack with length $2a$ in a plate of width, W	$K_I = \sigma\sqrt{W\tan\left(\frac{\pi a}{W}\right)}$
2 symmetrical edge cracks with length a each in a plate of total width, W	$K_I = \sigma\sqrt{W\left[\tan\left(\frac{\pi a}{W}\right) + 0.1\sin\left(\frac{2\pi a}{W}\right)\right]}$

Keratin the Method of Weight Function in the Calculation of Stress Intensity Factor

There are several existing methods to calculate the stress intensity factors; however, the methods require separate analysis for every load and geometry configuration. Bueckner (BUECKNER HF, 1970) and Rice (Rice, 1972) show the weight function method showing a simpler method to determine factors of stress intensity for any load and geometry configuration. The uniqueness is that the stress intensity factor for any loading system applied to the body can be determined easily with the weight function known if the weight function is known for a cracked body. This advantage leads no more additional derivations has to be done for each load system, hence simplify unnecessary steps to determine the factor of stress intensity.

In brief, the stress intensity for any loading system applied to the body can be calculated by using simple integration of the product of stress field, $\sigma(x)$ and weight function, $m(a, x)$ as shown in Equation 7, once the weight function for a cracked body is known. The weight function here to be known depends only on the component geometry.

$$K = \int_0^a \sigma(x) \cdot m(a, x) dx \tag{7}$$

$$m(a,x)=\frac{H}{K^*}\frac{\vartheta u}{\vartheta a} \tag{8}$$

The general form of the weight function is shown in Equation 8. The K* is the factor of stress intensity as reference for a loading case. H is the generalized Young's modulus, which equals to E for plane stress and E/(1-v2) for plain strain. The displacement, u in the equation means the displacement for the respective crack geometry in the crack body.

The only tedious step in using the method of weight function is to determine the weight function, m(a,x). Nevertheless, Glinka and Shen (Shen & Glinka, 1991) have discovered that the general weight function expression for a variety of geometrical configurations of cracked bodies with internal through and edge cracks of Mode I stress intensity factor. Equation 9 shows the weight function formula as found.

$$m(a,x)=\frac{1}{\sqrt{2\pi(a-x)}}\left[1+M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+M_2\left(1-\frac{x}{a}\right)^{1}+M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right] \tag{9}$$

The three parameters M1, M2 and M3 have to be determined before the weight function can be used. There are several methods which may be used depending on the amount of information available as explained by Shen and Glinka (Glinka & Shen, 1991). The most popular method is to base on two reference stress intensity factors and one based on the knowledge of the crack surface slope (Shen & Glinka, 1991). One thing to be noted here is that the edge cracks is the only interest crack profile used in this study, hence the three independent equations can be seen in Equation 10, 11 and 12.

$$K_{r1}=\int_0^a \sigma_{r1}(x)\frac{1}{\sqrt{2\pi(a-x)}}\left[1+M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+M_2\left(1-\frac{x}{a}\right)^{1}+M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]dx \tag{10}$$

$$K_{r2}=\int_0^a \sigma_{r2}(x)\frac{1}{\sqrt{2\pi(a-x)}}\left[1+M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+M_2\left(1-\frac{x}{a}\right)^{1}+M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]dx \tag{11}$$

$$\frac{\partial}{\partial \mathrm{x}}\left\{\frac{1}{\sqrt{2\pi(a-x)}}\left[1+M_1\left(1-\frac{x}{a}\right)^{\frac{1}{2}}+M_2\left(1-\frac{x}{a}\right)^{1}+M_3\left(1-\frac{x}{a}\right)^{\frac{3}{2}}\right]\right\}\Bigg|_{x=0} \tag{12}$$

Once three equations are form from the references, and then the unknown parameters M1, M2 and M3 can be determined by solving the set of equations simultaneously.

There are many past studies on determining the weight function for various crack geometry such as Wu-Carlsson Solution (Wu & Carlsson, 1991). The weight functions used for internal and external through cracks are based on stress intensity handbook (Dr S, 1998). The weight function used is discussed in Chapter 3 where all the related equations applied to determine the factor of stress intensity and compared with the finite element analysis.

RESEARCH METHODOLOGY

The general approach in this study is to determine the factor of stress intensity for non-welded and welded thick-wall cylinder. The methods of finite element and weight function are the two main approaches to find out the factor of stress intensity for the single edge crack in longitudinal butt-welded joint and compare with non-welded thick-wall cylinder.

Method of Finite Element

The method of finite element used will refer to computational engineering software: ABAQUS and the overall steps are given as below:

i. Construct a global model.
ii. Introduce a crack and non-crack global model
iii. Analyse the global model macroscopically.
iv. Extract the area of crack surface.
v. Construct the local model.
vi. Analyse the welded and non-welded crack on the local model.

There are three main procedures that need to perform in order to obtain the desired result from the ABAQUS. The three main procedures are pre-processing, simulation and post-processing. Figure 3 illustrates the overall procedures in ABAQUS.

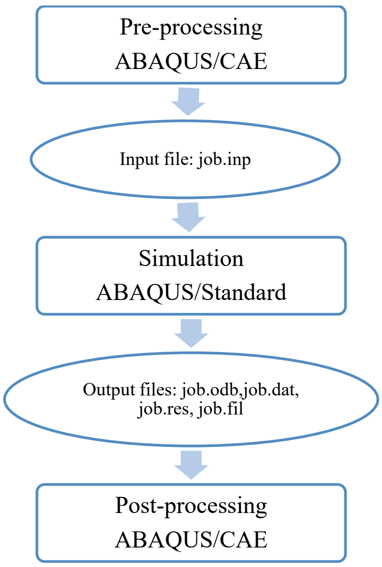

Figure 3. Three distinct procedures in ABAQUS

Method of Weight Function

The method of weight function is used for stress intensity factor in an external surface crack in a cylinder (W, 1997), it is shown in Equation 13.

$$K_I = \frac{1}{\sqrt{2\pi a}} \int_0^a \sigma(x) \sum_{i=1}^{i=4} f_i\left(a/t, \frac{R_i}{t}\right)\left(1 - \frac{x}{a}\right)^{i-\frac{3}{2}} dx \tag{13}$$

The stress state,σ(x) is to be taken normal to the prospective crack plane in a non-crack cylinder. The co-ordinate is described in Figure. The functions of geometry f (i=1 to 4) are all values and they are given in table form shown in Appendix B for the deepest point of the crack (D), which shown in Figure 4.

Equation 13 is also used to estimate stress intensity factor in the study of edge external crack at the butt-welded joint. The stress intensity factor in all cases determined by weight function method, Equation 13 is then compared with finite element results and other past studies.

The stress function, σ(x) was obtained from the non-crack thick-wall cylinder hoop stress distribution with respect to the prospective crack plane. The stress function is a polynomial function with high order of five with regression value minimum of 0.998. The polynomial function will be in the form as shown in Equation 14 which mi (i = 1 to 6) is the coefficients and x is the radial distance in the thick-wall cylinder.

$$\sigma(x) = m_1 x^5 + m_2 x^4 + m_3 x^3 + m_4 x^2 + m_5 x + m_6 \tag{14}$$

Thick-Wall Cylinder Models

The analysis models are drawn accordingly to the geometry profile and material properties in ABAQUS as shown in Figure 1.

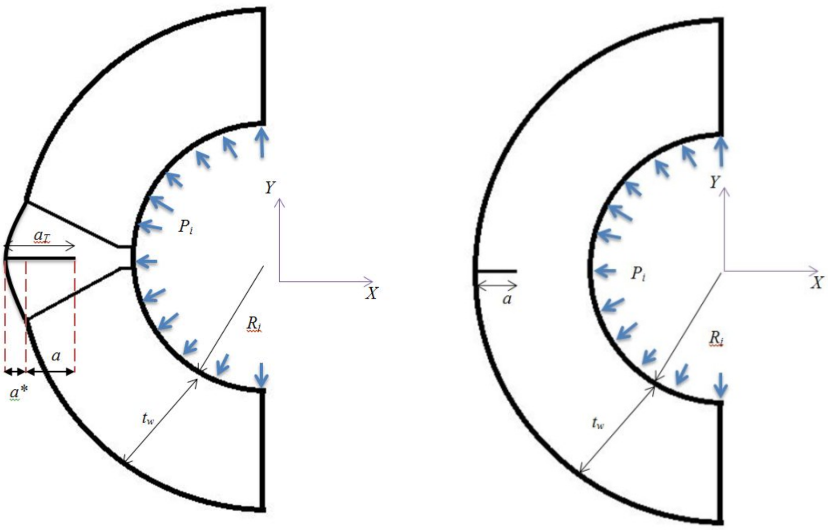

Figure 4. Single edge external surface crack in (a) longitudinal butt-welded joint thick-wall cylinder, (b) non-welded thick-wall cylinder

Method of Weight Function

The material properties for both weld material and thick-wall cylinder are the same. The material properties introduced is an isotropic, homogeneous and linear elastic material with elasticity modulus, E = 207 GPa and Poisson's Ratio, v = 0.33. The material is carbon steel which is a common and standard used in most construction of pressure vessels and pipes in industries. The most important aspect is the linear elastic property. This is the basic theory and assumption used throughout this study. Theoretically, the yield strength of the material is not considered in every case, thus it should have a linear relationship between stress and strain that obeys Hooke's Law with no proportional limit.

Input Parameters

Table 2 shows all the input parameters that are used in this modelling. A critical point to be considered here is that the wall thickness is selected to be 25 mm and is chosen to satisfy the definition of thick-wall cylinder which is the wall thickness must be greater than one-tenth of its radius (tw/Ri > 1/10). In addition, the geometry profile of the single butt-welded joint in the thick-wall cylinder is based on recommendation on welding codes and books [1-2, 8]. Figure 5 shows the butt-welded joint geometry profile used in this research. Such joint is commonly used in pipes and pressure vessels when only external surface welding is available which a lot of case is during field or on-site welding in order to fabricate or repair the equipment.

Table 2. Input Parameters in Thick-Wall Cylinder Static Analysis Modelling

Parameter	Value
Internal Pressure, P_i (MPa)	100
External Pressure, P_o (MPa)	0
Internal Radius, R_i (mm)	50
Outer Radius, R_o (mm)	75
Wall Thickness, t_w (mm)	25
Crack Depth Ratio, a/t_w	0.12, 0.20, 0.40, 0.247, 0.327, 0.527
Inner radius to wall thickness ratio, R_i/t_w	2

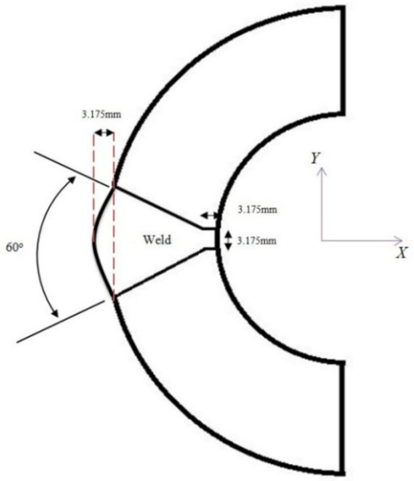

Figure 5. Single butt-welded joint in thick-wall cylinder

RESULTS AND DISCUSSION

Factor of Stress Intensity at External Straight Front Crack with Different Crack Ratios on Cylinder with Non-Welded Thick-Wall

Factors of stress intensity for various crack ratios at external crack in a thick-wall cylinder were shown in Figure 6. One thing is clear that the stress intensity factor magnitudes at external crack with various crack ratios are smaller compared to the case of internal crack. This is true as the thick-wall cylinder is subjected to uniform internal pressure only and the hoop stress acting on the crack line decreases from inner radius towards outer radius in radial direction.

The factor of stress intensity obtained from the ABAQUS finite element analysis is closely range with the method used in handbook. Surprisingly, the method used from C-C Ma, et al (Ma et al., 1994) showed that deviation occurred once the crack ratio is more than 0.2. It seems that the result obtained from this solution is not suitable even the suggestion for crack ratio ranges up to 0.8 and within $0.3 \le Ri/Ro \le 0.8$ in their paper.

The deviation of the obtained values using C-C Ma, et al solution may due to the assumption of the explicit form of weight function mechanical loading which based on the displacement of crack opening for an edge crack in plates with finite width and length. Even they did analysis for uniform internal and external pressure by using their own derived weight function, they did not validate to any known data or other methods. So in this study, their solution is not suitable to calculate straight front cracks in thick-wall cylinder when crack ratio is more than 0.2.

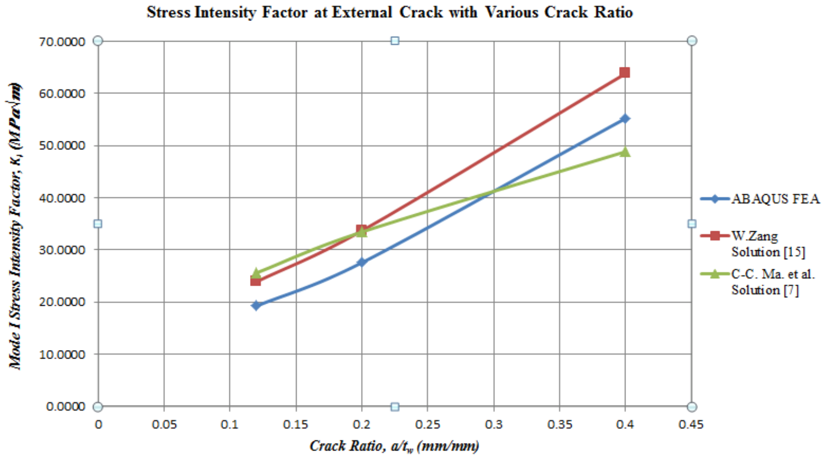

Figure 6. Stress intensity factor at external crack with various crack ratios on thick-wall cylinder

Nevertheless, the weight function method used which is Equation 13 could approximate the factor of stress intensity close to the result of the analysis of finite element. Since the weight function in Equation 13 has been developed suitable for external axial crack profile for hollow cylinder geometry, the stress intensity factor can be defined easily when the hoop stress distribution is identified.

Factor of Stress Intensity at External Straight Front Crack at Welded Joint with Different Crack Ratios

The factor of stress intensity for various crack ratios at external crack in a welded thick-wall cylinder were illustrated in Figure 7. One thing to be clear that the factor of stress intensity magnitudes for the external cracks at single butt-welded joint were based on crack ratios which are different with previous case due to the reason of comparing the same deepest crack point.

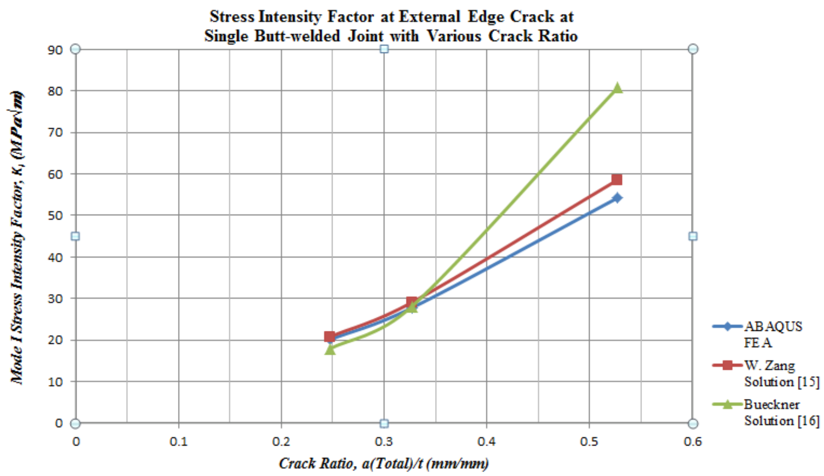

Figure 7. Stress intensity factor for external crack at single butt-welded joint with various crack ratios

Based on the results shown, it turns out to be that the weight function Equation 13 also is able to approximate stress intensity factor well compared to finite element analysis. This shows that only with different stress distribution profile used, Equation 13 is still able to perform satisfied with good accuracy even the weight function derived which based on geometry profile does not includes the weld joint geometry profile. This argues that the weld joint profile used in this study does not affect the stress intensity factor in thick-wall cylinder.

In using Bueckner solution (Grandt Jr., 1975) which is also a weight function approach, it approximates the stress intensity factor quite well but when the crack ratio is more than 0.35, it starts to deviate from the finite element results which also mentioned in his paper that the solution can only be used for very small cracks.

Comparison of Stress Intensity Factor between Non-welded Thick-wall Cylinder and Welded Thick-wall Cylinder

Because of the same depth of the deepest crack point used for both welded and non-welded external crack in thick-wall cylinder, we can see the comparison in terms of factor of stress intensity. As shown in Figure 8, the magnitude of stress intensity factors for both cases for all crack ratios are the same. This can be clarified that despite the existing of the weld joint, it does not affect the stress intensity factors at the crack region which determined by the analysis of finite element. In the case of using the weight function Equation 13, the result also shows similar trend with satisfied accuracy as shown in Figure 9.

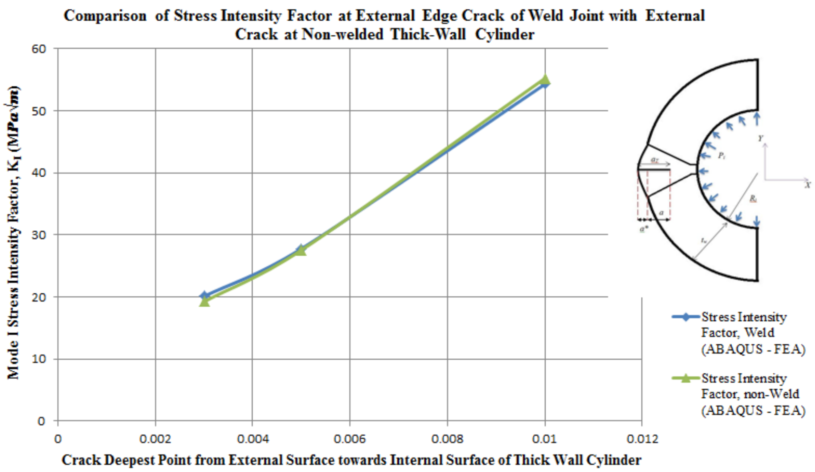

Figure 8. Comparison of stress intensity factor at external edge crack of weld joint with external crack at non-welded thick-wall cylinder

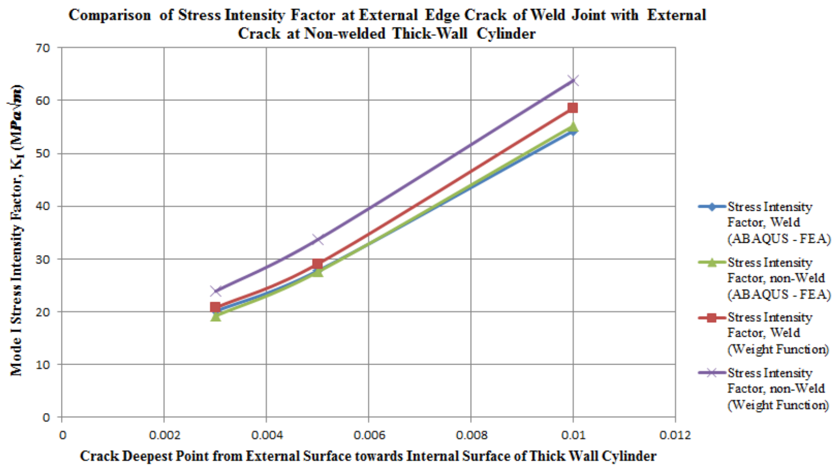

Figure 9. Comparison of stress intensity factor at external edge crack of weld joint with external crack at non-welded thick-wall cylinder including weight function approach

Discussion

As shown in all results, the method of weight function is worth-using to determine the factor of stress intensity in the thick-wall cylinder whether it is non-welded or with welded joint. Surprisingly in the case of the single butt-welded joint in thick-wall cylinder, the weight function Equation 13 which uses for non-welded thick-wall cylinder with external surface axial crack can also be used. Despite that the weight function depends only on geometry and in this case, a new weight function for the welded thick-wall cylinder should be change due to the change in geometry, Equation 13 could still calculate well the factor of stress intensity when compared to finite element solution.

The only change in the weight function method used is the stress distribution profile. In the case of non-welded thick-wall cylinder, the stress distribution is based on the hoop stress obtained from non-crack thick-wall cylinder, whereas for the welded thick-wall cylinder case, the stress distribution profile is based on the existing weld joint with no crack. With the stress function known, simple integration can be done for Equation 13. Thus, it shows that the method of weight function can be convenient to use as long as the weight function is known for the particular crack geometry and the loading system can be varies.

Since the weight function requires the stress function for a non-cracked geometry to calculate the stress intensity factor, polynomial fits were used to obtain stress functions from finite element analysis results. The polynomial equations used in all cases were up to order of five with regression value to be minimum 0.998 as shown in Equation 14. The reason of choosing higher order of polynomial with high regression value is to make sure that the polynomial equation fits with the stress profile. This means that the equation obtained represents the stress function with high accuracy.

In short, the weight function in this study has been used effectively to transform the hoop stress distribution profile in no crack conditions into hoop stress distribution profile in crack conditions.

Another point to be noted here that despite the weld joint profile does not affects the stress intensity factor in this study, it does not generally concludes that weld joint profile does not affect for all cases in any geometry. There are many variables such as residual stress due to welding process, post weld heat treatment weld joint throat size and weld material and mechanical properties and etc. which affect the stress intensity factor. In addition, the weld joint profile is assumed to be ideal case and the weld material does not have any effect on the parent metal. Thus a more in-depth study is needed to investigate the effect of weld joint profile in thick-wall cylinder.

Despite having similar stress intensity magnitude in both welded and non-welded case in this study, the stress concentration arose at the corner of the butt-welded joint cannot be neglected. Therefore, surface finishing or trimming the excessive weld throat is needed because not only it can remove surface flaws, it can also eliminate corners which act as stress raiser that contributes to crack propagation. This can be seen clearly in the stress distribution obtained from ABAQUS in Figure 10.

The phenomena of the crack length increases causing the stress concentration to increase can also be explained with the energy-balance approach. When a crack exists in the thick-wall cylinder with certain crack depth, the crack line region adjacent to the free surface is unloaded by hoop stress, thus its strain energy is released. As the crack increases, additional quantity of strain energy is released from the newly-unloaded material near the crack. The unloaded material contains zero stress while the remaining structure continues to feel the overall applied hoop stress. Therefore, the remaining structure consists of higher stress distribution especially at the crack tip.

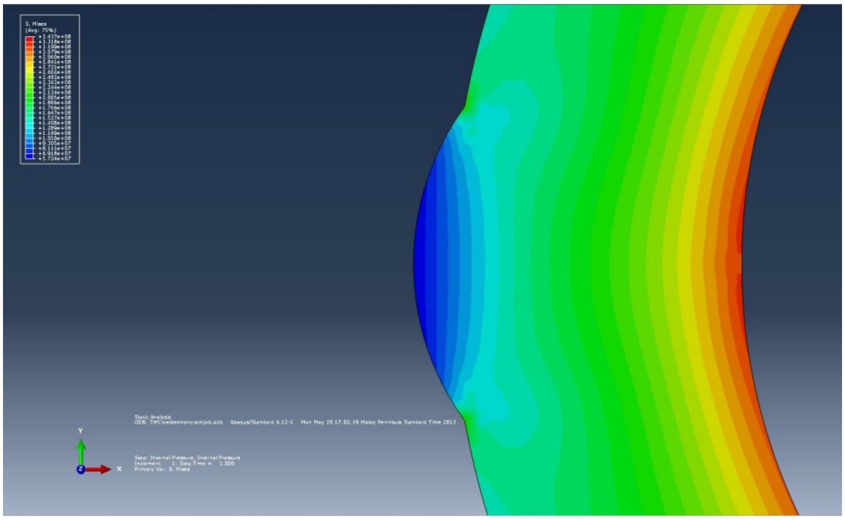

Figure 10. Stress risers at the corner of the butt-welded joint in thick-wall cylinder

The reason of using stress intensity factor is because it can determine the driving force of a crack growth. To relate with stress concentration factor, stress intensity factor take the crack geometry into account and depicts how the crack profile will affect the stress at the tip region of the crack, thus it shows the driving force for crack to propagate. Stress concentration offers notions which areas that crack may grow and stress intensity notify the driving force for a crack to propagate

CONCLUSION

i. The result from the finite element analysis was compared with other solutions such as weight function method and empirical method. Results shows that the weight function method is able to calculate the factor of stress intensity with satisfied accuracy compared to finite element solutions. It also shows that method of weight function is a suitable way to define the factor of stress intensity for thick-wall cylinder.

ii. The weld joint profile shows no effect in stress intensity factor calculation in thick-wall cylinder in this study. Results show that the magnitudes of the factor of stress intensity are quite similar with both non-welded and welded thick-wall cylinder. Nevertheless, this does not mean that the weld joint profile has no effect in all cases of thick-wall cylinder. Suggestions have been made in order to have a more in-depth study on the effect weld joint profile in stress intensity factor thick-wall cylinder. In addition, both finite element solution and weight function method are able to determine the stress intensity factor for the welded thick-wall cylinder. The method of weight function used to determine the stress intensity factor for the weld joint in thick-wall cylinder is Equation 13 described by W. Zang paper [15]. It shows that with the stress distribution profile and weight function being known, the stress intensity factor is able to be easily determined without any repeating steps for various crack ratios.

iii. Surface finishing or trimming the excessive part of the butt-welded joint may seem to be not required from fracture mechanics point of view since the factors of stress intensity obtained shows similar magnitude. Nevertheless, it is good to perform such practice from stress raiser point of view since it can eliminate stress concentration areas to prevent crack or surface flaws to occur.

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