Using mathematical model to design and sizing of pumping water system

Luqman Raji *, Gajawa, Y, A. B, Aji, Kasni Sumeru

Department of Mechanical Engineering Technology, Federal Polytechnic Mubi, 650272 Adamawa State, Nigeria
Department of Agricultural and Bio-Environmental Engineering Technology, Adamawa State College of Agriculture Ganye, 641113 Adamawa State, Nigeria
Department of Agricultural and Bio-Environmental Engineering Technology, Adamawa State College of Agriculture Ganye, 641113 Adamawa State, Nigeria
Center for Entrepreneurship Development, Federal Polytechnic Mubi, 650272 Adamawa State, Nigeria
Department of Refrigeration and Air Conditioning Engineering, Politeknik Negeri Bandung, Bandung 40559, Indonesia

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ABSTRACT
The pumping of water worldwide is mainly dependent on conventional sources of electricity such as grid electricity, diesel generators, or gas or coal-based electricity. The pump's job in any pumping system is to create sufficient pressure to overcome the operating pressure of the system to transport fluid (water) at a specific flow rate. The complete flow system must be considered while choosing a suitable pipe. Pipe length, fittings, pipe size, change in liquid elevation, and pressure on the liquid surface must all be considered. This component determines the needed flow rate through the pumping system and the system operating pressure. Agricultural output in developing countries is highly dependent on rainfall, and water scarcity during the hot summer months has a negative impact. Summer, on the other hand, has the most solar radiation. Therefore, more water can be pumped to meet the increased demand. The use of energy to pump water in urban areas also depends on electricity. Photovoltaic (PV) pumping systems for water supply have many applications in rural, urban, community, industrial, and educational organizations.

INTRODUCTION
Water pumping is mostly depended on conventional energy or diesel-generated electricity around the world. Solar water pumping reduces the need for diesel, gas, or coal-fired power. Water pumping systems based on diesel or propane consume not only expensive fuels, but also produce noise and pollutants. A diesel pump has a 2–4 times higher initial cost, operation and maintenance costs, and replacement costs than PV pump. Solar pumping systems are
environmentally benign, low-maintenance, and require no fuel [1]. PV pumping is one of the most promising applications of solar energy. With the exception that it is powered by Solar energy, the technology is comparable to any other typical water pumping system. Due to a scarcity of energy and rising diesel prices, PV water pumping system. Due to a scarcity of energy and rising diesel prices, PV water pumping has been increasing popular in recent years. Pumped water flow rates are determined by incident solar energy and the size of the PV array. When compared to traditional pumping systems, a correctly constructed PV system leads to significant long-term cost savings. Tanks can also be used to store water instead of batteries, which are needed for electricity storage [2].

In a solar photovoltaic water pumping system, Chandel et. Al [3] investigated numerical stimulation and design parameters. In Nigeria, they looked at the relationship between array power and borehole depth. The collecting of data from the meteorological department is necessary for efficient water pumping. Abu-Aligah [4] reported the necessary steps and key components that needed in design and build a pump. Girma et al. [5] reported that the drinking water supply in Ethiopia is 91.5% in urban and 62% in rural area and concluded that till now some Ethiopians are lack access to safe and reliable source of drinking water. The performance of a photovoltaic-based submersible water pump was studied by Lal et al. [6]. In their research, they discovered that the highest discharge was attained at 12 p.m. for a 2 hp DC motor with with 225 W panels and a power output of 75 to 85 W/m². Odeh et al. [7] evaluated the economic viability of solar and diesel water pumping systems for different system sizes in the range of 2.8-15 kWp based on real data and three years operational experience of eight installations. The concept of minimizing the cost of a water unit by anticipating demand patterns, sizing storage tanks, and using low-pumping head wells is investigated. A mismatch between water demand and supply has a substantial impact on the economic feasibility of PV pumping systems, according to the study, and hence must be thoroughly explored.

Haddi et al. [8] stated that pumping water is the best solution for irrigation of agricultural land and drinking water in the Saharan regions. Similarly, Girma et al., [5] stated that the traditional water pumping systems is diesel, gasoline engines which has been used for decade but due to cost of fuel, transportation problem, lack of skilled personnel makes this conventional water pumping system unreliable and expensive for many communities. Chandel et al. [3] reported on review of water pumping system technology for irrigation and community drinking water supplies. Their main objective of the paper is to present a comprehensive up to date literature review of solar water pump and also concluded that solar water pumping is more economically viable when compared with electricity or diesel based Hadid power supply system for irrigation and water supplied in rural, urban and remote regions. The dynamic modelling of a system was carried out by Biswas & Iqbal [9] in order to extraction of ground water through an alternative source of energy, the authors design the system based on the existing data obtained in Lalmonirh at, Bangladesh.

Hossain et al. [10] reported that out of 150 pumps existed in Bangladesh in year 2010, out of this pump only 65% of it is used for drinking water supplying and remaining one is used for irrigation purpose. Abu-Aligah [4] stated that photovoltaic (PV) pumping system is capital in tensive because it requires water storage tank for cloudy weather and skilled personnel. Algorithm developed was carried out by Loxsom & Durongkaveroi [11] to estimate the monthly
water discharge of a battery less PV water pumping rate and its isolation. Abul-Hasnat et al. [12] worked on hybrid system for water pumping and also concluded that hybrid Solar irrigation is better when comparison with solar standalone system in terms of payback period. Jamil et al. [13] demonstrated a $20,000 solar water pumping system to meet the water needs of an academic institution in New Delhi, India. The results of a technoeconomic analysis of a PV based water pumping system are compared to those of an existing system. The proposed system has a four-year payback period and a 20year life, allowing it to supply water for free for 16 years, which is a major cost reduction and eliminates the grid dependency of current electricity based and diesel water pumping systems [3].

Hamidat [14] conducted an electrical and hydraulic performance analysis of a surface centrifugal pump for three PV arrays and multiple total dynamic heads in Algeria, concluding that the average annual flow rate for 14.5 m³/head is around 60 m³/day and the cost of water provided is US $0.04/m³. For the socio-economic development of the region, the study advises deploying surface PV pumps to supply water in distant Sahara locations. Gad [15] developed a methodology for the performance of a direct coupled PV water pumping system in South Sinai using a computer simulation tool (Egypt). He used program to simulate to simulate the hourly performance throughout the year and concluded that the system is capable to pump 24.06 L/day, 21.471 L/day and 12.121 L/day in summer, equinoxes and winter days respectively. Studied and analysis of performance of PV-Powered DC motor couple with a centrifugal pump at different solar intensities and temperature was experimental by Muluken et al. [16] when they compared the result obtained with calculated values. The authors concluded that their results had a good agreement with the electro-mechanical system characteristics. Design and Evaluation of solar water pumping system presented in sultanate of Oman. Authors stated that there are two option to pump water: solar water pump and conventional pump system i.e. electricity or diesel generators for water pumping but the later solution is very costly over the long run period and also not environmentally friendly at all [17]. Muluken et al. [16] presented a cost and reliability by comparing solar and diesel-powered pumps in Dangila, Ethiopia. This study used design and sizing water pumping system through mathematical model approach Federal Polytechnic Mubi main Campus quarter in Mubi, Adamawa State, Nigeria. The analysis also determined capacity of photovoltaic panel and inverter to be used without using battery in order to supply adequate quality of water.

**MATERIALS AND METHODS**

**Area of Study Site**

Mubi is the major commercial center in Adamawa state Nigeria. Federal Polytechnic Mubi (FPM) situates in the North Eastern Zone of Nigeria with 580m above ordnance datum (AOD). The geographical location of Mubi is located between latitude 10.27° N and 13.28°E with tropical wet and dry climate. According to Raji et al. [18], Mubi town has three climate/weather conditions where Dry season last for five months (November to March) while wet season span is seven months (April to October). It is headquartered of Mubi North local government area.
with land area of 4728 km² and a population of 759045 in 2003 (1991 projected census figure) as presented by Adebayo [19]. The study area map was described in Figure 1.

![Map of study area from google map](image)

**Figure 1. Map of study area from google map**

**Source of Water**

Water supply is usually from a river, a pond, borehole (deep drilled well), spring or reservoir. The water is usually pumped from the source unto an overhead tank (receiving tank) or an arrangement is used to transport water from a river/borehole to a water treatment plant, where it is treated before being fed into supply network pipe. According to Chandel et al. [3], if the pumping rate of water is quicker than the recharging rate of the reservoir will dry out, damaging the pump. In fact, the design of a water is determined by the amount of the reservoir, the rate recharge, and the cost.

Surface water sources vary with the seasons, with the amount and quality of water being low during the summer when it is most needed.

**Water Storage**

The water storage system size and cost are normally depended on the amount of water required per day. Alternative current (AC) pumping systems connected to a national grid power are designed to run on demand with a specified flow rate. Which is not like solar pumping system that designed to provide a certain quantity of water per day. ie pumping water during the sunlight hours and store it in a tank so that the user can have access to it when needed. In the report of AECOM 2017 [20] the water level in the source varies but it's discharge level in the storage tank remains constant as the water is discharged from a point above the water level and any storage tank should large enough to hold a water for three to five days of daily output.
Estimation of Water Require

The daily requirement of water is the total of all water required during a day i.e., 24 hours period. Hadidi et al. [8] stated that, for a tropical region water requirement may be Humasis: 5000 to 1000 m³/day minimum; normal living conditions 30 L/day. Animals: sheep and goats: 5000 m³ (5 L)/day, Horse; 40000 m³/day (40 L/day) Agricultural irrigation: Rice 100 m³/ha/day; sugar can: 65 m³/ha/day and cotton: 55 m³/ha/day.

In this research data was collected at work department, FPM main campus, Mubi as following. Block A has 4 apartment, block B has 38 apartments block J is 8 apartment block K is 6 rooms tank in each department which is equal to 44 and tanks in rectory & bursary is 96000 L/day, 14000 L/day, 11000 L/day and 2500 L/day respectively, which their total is 123,5000 L/day (50.146 m³/hour).

The Operating Pressure

The operating pressure of a pumped system in this study is calculated in standard international unit (SI Unit) of meters (m). In order to maintain dimensional consistency in the calculation, all pressure values used in the computations are transformed from kPa to m, i.e., 1 kPa = 0.102 m as measured using a water filled U tube manometer.

Estimation of Total Dynamic Head

The schematic diagram of pumping water arrangement is shown in Figure 2. Total Dynamic Head \( THD \) is the sum suction (static) head \( H_{sh} \) Dynamic Head \( H_{dh} \) and difference between pressure on the surface of the water in the receiving storage tank \( P_{rs} \) in meter and pressure on the surface of the water in reservoir \( P_{sr} \) is also in meter (m) as given in equation (1).

\[
THD = H_{sh} + H_{dh} + (P_{rs} - P_{sr})
\]  

In Fact, the atmospheric pressure varies with height and the change that occur in pressure over the pumping height is normally negligible because it is often so small. In this research
work, the change in pressure over the height from reservoir to the receiving storage tank is too small i.e., negligible, therefore \( P_r - P_s = 0 \). Hence equation (1) would become equation (2).

\[
THD = H_{sh} + H_{dh} \quad (2)
\]

**The static Head**

The static Head \((H_{sh})\) is normally a physical change that occur between elevation surface of reservoir and the point of discharge into the receiving tank. As the \(H_{sh}\) of the system vary between a maximum and minimum value, the water level in the reservoir also varies. The static Head maximum \((H_{smax})\) and static head minimum \((H_{smin})\). Hence, express in equation (3) and (4).

\[
H_{smax} = D_L + R_{twl} \quad (3)
\]

\[
H_{smin} = D_L + R_{twl} \quad (4)
\]

Where, DL is Discharge level, \(R_{twl}\) Reservoir at \(T_{Twl}\), \(R_{twl}\) is Reservoir at \(T_{Twl}\), \(T_{Twl}\) is Top water level of reservoir and \(L_{twl}\) is Bottom water level of reservoir.

The analysis assumes that the discharge point is at a height of 150.5 meters above ordinance datum (AOD) and that the reservoir level varies between 114.2 and 113.3 meters AOD, then

\[
H_{smax} = 120.5 - 111.3 = 9.2 m \\
H_{smin} = 120.5 - 114.2 = 6.3 m
\]

As a matter of fact, the TDH will also have maximum and minimum value because of the variation that occur in the \(H_{sh}\).

**The Dynamic Head**

The dynamic head \((H_{dh})\) is always occurred as a result of friction within the system. Darcy weisbach equation given in equation 5 is usually used to calculate \(H_{dh}\).

\[
H_{dh} = \frac{k \nu^2}{2g} = 0.02 m
\]

1. **Parameter Calculated**

Velocity in the pipe \((\nu)\) is calculated by dividing the flow rate \((Q)\) through the pipe \((m^3/s)\) to pipe cross sectional area \((Ap)\) \((m^2)\). Here the pump is required to pass forward a flow of water
(5.146 m³/hr) to receiving tank and the system is using 0.6m diameter pipe \((D)\) to supply the water around the campus tanks. The \(V\) and \(A\) can be calculated by equation (6) and (7).

\[
A = \frac{\pi D^2}{4} = 0.05 m^2 \tag{6}
\]

\[
V = \frac{Q}{A} = 0.03 m s^{-1} \tag{7}
\]

2. Loss of Coefficient

The loss of coefficient \((K_f)\) is consisted of two parameters such as fitting of each material \((K_f)\) used in the pipe work of the system to pump the water from reservoir to the receiving tank and the straight lengths of pipe \((K_p)\) used within the system as given in equation (8).

\[
K_{ft} = K_f + K_p \tag{8}
\]

A total \(K_{ft}\) value was calculated by adding all the \(K_f\) value for each individual fitting used within the system. The calculation of all \(K_f\) for the system under consideration is shown in Table 1.

<table>
<thead>
<tr>
<th>S/No</th>
<th>Fitting items</th>
<th>Number of items used</th>
<th>(K_f) Value</th>
<th>Total item</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pipe entrance (bell mouth)</td>
<td>1</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>90° bend (short radius)</td>
<td>4</td>
<td>0.75</td>
<td>3.00</td>
</tr>
<tr>
<td>3</td>
<td>45° bend (short radius)</td>
<td>1</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>4</td>
<td>Butterfly Valve (full open)</td>
<td>1</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>5</td>
<td>Non return value</td>
<td>1</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>6</td>
<td>Bellmouth outlet</td>
<td>1</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>Total (K_f) Value</td>
<td></td>
<td></td>
<td>4.85</td>
</tr>
</tbody>
</table>

As it was revealed in the table 1, the total \(K_{ft}\) for the system under consideration is 4.85. Hence, the \(K_p\) can be determined through equation (9) – (11).

\[
K_p = \frac{fL}{D} = 35.90 \tag{9}
\]

\[
F = \frac{0.25}{\log \left( \frac{K_f}{\frac{5.74}{R_e^{10}}} \right)} = 0.038 \tag{10}
\]

\[
R_e = \frac{VD}{u} = 5.816 \times 10^3 \tag{11}
\]
Where, $F$ is friction coefficient, $L$ is pipe length (m), $D$ is pipe diameter (m), $K_{rf}$ is roughness factor (-) and $re$ is Reynolds number (-).

A modified version of the Colebrook white equation was used to calculate “$f$” as given in equation 10. The pipe $K_{rf}$ roughness factor used in the system was obtained from standard tables which depend on the type of material used to make the pipe, including any internal coatings and the internal condition of the pipeline i.e., good normal or poor (Miller, 1984). For any flow in pipeline, Reynolds number can be determined as in equation 11, which is as dimensionless quantity that normally associated with the smoothness of flow of fluid and relating to the energy absorbed within the fluid as it flows in the pipe. The total pipe length used in 250m sand it pipes roughness factor is 0.3mm with kinematic viscosity of water is $1.31\times10^{-6}$ m²/s i.e., ms⁻¹. The parameter was substitute in equation 9 – 11 to obtain $K_p$ and determine $K_{ft}$ value for the system express in equation 8.

$$H_{S\text{max}} = 9.2 + H_{dh} = 9.202\text{m}$$

$$H_{S\text{min}} = 6.3 + H_{dh} = 6.302\text{m}$$

**Selection of A Pump**

A device for sucking and discharging a fluid is pump which is suitable for raising low water flow at high pressures. Hadid et al. (2016) state the two main type of pump such as centrifugal pump and positive displacement pumps. In order to select the best pump speed, need to be reduced to achieve the required flow rate at the Rtwl and its speed required which was determined using the affinity laws as in questions 14 and 15.

$$Q_a N_b = Q_b N_a$$

$$H_a (N_b)^2 = H_b (N_a)^2$$

Where $Q_{a.b}$ is the flow through the pipe (m³/s), $N_a, b$ is the shaft speed (rpm) and $H_{a,b}$ is head (m).

**Pump and Motor Size Power**

The hydraulic power of the pump and induction motor power model used is given in equation 16 (Biswas & Iqbal, 2018). The induction motor pump efficiency ($\eta$) considered is 90% for this pump, the maximum head $H_{\text{max}}$ and a flow is 10.39 m and 5.146 m³hr⁻¹ (m³s⁻¹) respectively.

$$H_a (N_b)^2 = H_b (N_a)^2$$

Where, $P$ is power (W), $\rho$ is Density of water (kgm⁻³) i.e 1000kgm⁻³, $A$ is volumetric flow of water through the pump (m³s⁻¹) $H_{\text{max}}$ is minimum head obtained during the flow of 5.146 m³/s.
Inverter Required for The System

Power conditioning unit is the order name of inverter which is the heart of any solar system. The application of inverter is to converting DC power to AC power in PV system. Solar pump inverter chosen is SPI 250-4T-37B inverter model and its operating DC input voltage is within the range of 450-820VDC with output voltage of 300V AC/50Hz frequency. It has 90% of efficiency. Rated current is 15 A. The inverter power \( P_i \) required given in equation (17).

\[
P_i = \frac{\text{Pump power required}}{\text{Inverter efficiency}} = 1500W
\]

Since, the \( P_i \) required is 1430 W, therefore the solar power required is also 1500 W.

1. Number of panels required

The number of panels depend on the output of solar panel consider. Here we consider 150 W, 610 V and 1.64 A panel.

\[
\text{Number of panel} = \frac{\text{Solar power required}}{\text{Output power of panel}} = 10\text{panel}
\]

\[
\text{Number of parallel strings} = \frac{\text{Rated}}{\text{Output power of panel}} = 10\text{panel}
\]

\[
\text{Number of panel in each string} = \frac{\text{Number of panel}}{\text{Number of parallel string}} = 1\text{panel in each string}
\]

Power produced = 150(10)(1) =1500W

RESULT AND DISCUSSION

The purpose of employed the mathematical approach for this study was to determine the feasibility studies at the site where the pump is going to be installed in order to supply the adequate water request/ needed at a particular time in the main campus of Federal Polytechnic Mubi. It has been reported by Raji et al. [18] that weather conditions usually varied from one place to another just like solar radiation, relative humidity, pressure and temperature and further more stated that solar radiation of North varies from 3.43 kWm²/day to 7.3 kw/m²/day. In this particular study annual solar radiation was (5.74 kWh/m²/day). The results revealed has is maximum at 9.2 m and minimum at 6.3 m with 0.02 m of \( T_{dh} \). Therefore, the \( T_{THD} \) will be maximum at 9.202 m and 6.302 m of pressure.

The pump is required to pass forward a flow of water at 50.146 m³/hr. to the receiving tank and from there, 0.6m diameter pipe will be used to supply the water to other campus tanks (to user tanks). The computation shows the loss of coefficient \( (K_f) \) in the pipe to be 40.76 with
friction coefficient \( f \) of 0.038. The model computed shows the hydraulic power of the pump and shows induction motor power to be 1360 W. The model revealed the inverter required for this study is 1500 W. With number of panels required is 3 panels. Number of parallel string and number of panels in each string is 10 and 1 panel respectively.

**CONCLUSION AND RECOMMENDATION**

In this paper, a novel full of centralized water supply to varying quarter was carried out at FPM main campus in Adamawa state, Nigeria, using a mathematics model approach. Some of the parameters used as inputs to the model were total dynamic head \( THD \), flow rate, and loss of coefficient, which provided adequate pump and motor size power to pump water to varying quarters. When we chose the SPI150-4T-37B inverter type, which has a 90% efficiency, the result showed that the proposed water plant installation required 1500 W solar electricity. As a result, one of the inverters will be used to connect the devices. One panel and one inverter were also discovered in the results. Future research is needed to confirm the conclusions validate from this study. We recommend that the management find a way to establish the planned adequate water supply to the main campus, FPM Mubi, Adamawa State, Nigeria. Second, during future installations, the engineer team should consider the complete cost of the system, including the battery, as well as the payback period of the entire system.

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