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The Effect of Inclusion on Crack Propagation Using Extended Finite Element Method

[a]Aji Pratama Rendragraha, [b]Luthfi Muhammad Mauludin*

[a]Institut für Statik und Dynamik der Tragwerke, Technische Universität Dresden, Germany
[b]Civil Engineering Department, Politeknik Negeri Bandung, Bandung 40559, Indonesia

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ABSTRACT
Numerical simulation is developed to investigate the effect of inclusion on crack propagation. In this study, the crack growth is modeled using extended finite element method (XFEM). Two-dimensional rectangular plate with single inclusion embedded off-centered is modeled. The specimen is subjected to uniaxial tension. The dimensions of the specimen are 40 mm x 80 mm and the radius of the inclusion is 10 mm. The specimen is pre-cracked with the length of an edge crack is 5 mm. The motion of the crack is modeled by XFEM based on traction-separation cohesive behavior for 2D mixed mode problem. In addition, enrichment procedure is used to implicitly determine predefined crack in XFEM framework. Two different inclusions, which are soft and hard inclusions, are considered on crack propagation scheme. The effects of soft and hard inclusions on crack propagation are studied and observed. The results showed that the trajectory of crack highly depends on inclusion inside the material. In the case of soft inclusion, propagation of the crack tended to approach the inclusion. Whereas in the case of hard inclusion, crack trajectory tended to move away from the inclusion. The mismatch of elastic modulus between inclusion and surrounded materials has significant effect on propagation of crack.

KEYWORDS
Crack propagation
XFEM
Level set
Inclusion

INTRODUCTION

The crack propagation problem in finite element method is very challenging task, since it has to be modified in such way to accommodate the discontinuities caused by presence of cracks, voids or inclusions. The traditional finite element method is not suited for modeling crack propagation because at each increment of crack growth, the surrounding domains of the crack tip have to be remeshed in order to represent updated crack geometry precisely.

Over the past decade, some researchers had given so much effort working on fracture. Various methods and computational techniques have been developed to investigate fracture in brittle and quasi-brittle [1]. For example, meshfree technique to model discrete crack was proposed by Rabczuk and Belytschko [2], [3]. Random orientation of crack's angle and

*Corresponding Author: luthfi-mm@polban.ac.id

arrangement can be practically solved with this method. Rabczuk and Zi [4] investigated crack propagation for both of static and dynamic problem with a meshfree method. Areias and Rabczuk [5] introduced cohesive law within thin shell models (Kirchoff-Love) in bending application problem.

The extended finite element method (XFEM) to study crack propagation problem has received more attention from some researchers [6]–[8]. Node-based smoothed combined with extended finite element method has been developed by Vu-Bac et al [9]. The stress singularity at the crack tip can be circumvented with this technique. Phase-field using thin (Kirchoff-Love) shells combined with local maximum-entropy (LME) meshfree technique has been introduced by Amiri et al [10], while extended local maximum-entropy (XLME) complemented with enrichment function has been further developed [11].

LITERATURE REVIEW

The cohesive zone modeling to simulate fracture process in rock materials has been presented by Gui et al. [12]. They considered both of elastic and inelastic displacements to control the fracture. Characteristic of hydraulic fracture in permeable porous materials has been investigated by considering continuous and discontinuous pressure [13]. The new computational algorithms have been proposed for fracture in brittle and ductile materials based on edge rotations at the crack front [14], [15] and injection of continuum elements [16]. Areias et al. [17] developed local remeshing method based on phase-field to study fracture behavior of shells and plates.

Another computational algorithms to simulate crack growth in the materials have been developed using a modified screened poisson [18], tetrahedral refinement mesh based on edge division [19], and dual-horizon perydinamics [20], [21]. Hamdia et al. [22], [23] proposed an artificial neural network (ANN) along with Bayesian method to predict fracture toughness of polymer nanocomposites. Talebi et al. [24], [25] investigated fracture process using concurrent multiscale technique coupled with molecular dynamics and extended finite element.

The multiscale method incorporating phantom node [26], [27] and an equivalent coarse grained model [28] to simulate crack propagation have also been developed. The coarse-grained (CG) model was also used to predict elastic properties of carbon nanotube (CNT) through molecular interaction between polymer and nanotubes [29], [30]. In addition, other computational techniques have also been presented, such as the particle method [31]–[33], the cohesive crack [34], [35], the meshfree [36]–[40], the partition of unity [41], and the isogeometric analysis [42]–[48].

Recently, computational modelings of fracture in self-healing materials are getting so much attention [49], [50]. Mauludin et al. [51] developed computational model of fracture in encapsulation-based self-healing concrete. They introduced an algorithm to generate random microstructures inside concrete materials. The zero thickness cohesive elements were used to represent potential cracks along the boundaries of elements. The effects of volume fraction of capsules on load carrying capacity of concrete and probability of capsules to be fractured were investigated. The role of interfacial strength between the capsule and the mortar matrix was also studied by Mauludin et al. [52], [53]. They modelled the interaction between a single circular capsule with different core-shell thickness ratio and a pre-defined crack. The cohesive elements with bilinear traction-separation law were used to simulate crack propagation. The effects of interfacial fracture strength between the capsule and the mortar matrix on fracture probability and load carrying capacity were evaluated.

In this study, the fracture characteristic on how an approaching crack interacts with the presence of inclusion is investigated numerically. The crucial aspect in this interaction is the change of crack path due to the existence of inclusion. A rectangular plate with an edge crack and single inclusion is modeled under uniaxial tension. The two types of inclusions are considered here, which are soft and hard inclusions. The extended finite element method (XFEM) is used to simulate crack growth inside specimen. The objective of this study is to investigate the role of soft and hard inclusions on crack trajectory.

RESEARCH METHOD

Geometry of the model

In this section, the interaction of crack propagation in the existence of inclusions both of soft and hard inclusions is investigated. Two-dimensional rectangular plate with single inclusion embedded off-centered is modeled. The specimen is subjected to uniaxial tension. The dimensions of the specimen are 40 mm x 80 mm and the radius of the inclusion is 10 mm. The specimen is pre-cracked with the length of an edge crack is 5 mm. The schematic of the specimen along with the boundary and location of the inclusion is illustrated in Figure 1.

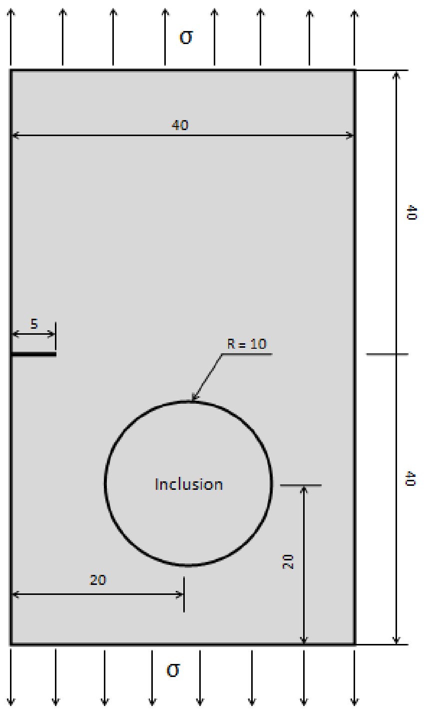


Figure 1. Schematic view of crack-inclusion problem

Two types of inclusions are considered in this study based on the ratio of Young's modulus between the plate and the inclusion. When the Young's modulus of inclusion is higher than the plate, it calls hard inclusion. Whereas the Young's modulus of inclusion is lower than the plate, it calls soft inclusion. Let R is the ratio of Young's modulus between the plate and the inclusion,

$$R = \frac{E_{plate}}{E_{inclusion}} \tag{1}$$

and the Poisson's ratio between the plate and the inclusion is assumed to be constant. The numerical simulations are conducted for two cases, that is (1) soft inclusion with R = 10, and (2) hard inclusion with R = 0.1.
The simulations of problems are conducted using ABAQUS 16.4 in plain strain condition and linear elastic is considered for material characteristic. In purpose for this simulation, the material properties for Young's modulus and Poisson's ratio of the plate are employed as E = 1.0e4 (KPa) and ν = 0.33, respectively. Whereas the material properties for the inclusion follow the rules based on the ratio of Young's modulus as explained previously. The specimens are discretized by applying 0.3 mm global mesh size to generate 4138 quadratic elements.

Extended finite element method

Modeling fracture process along with propagation of crack in the materials is not an easy task. Even though there were a lot of researches have been made to describe this complex process, but there is no exact method able to simulate all nature's aspects of fracture and describe it in detail. The extended finite element method (XFEM) in ABAQUS 6.14 is used in this study to simulate crack propagation in the presence of inclusion. This extension of conventional finite element method was first presented by Belytschko and Black [54].

The concept is based on the partition of unity which incorporated local enrichment function into finite element calculations. Crack propagation simulations with XFEM do not need path definition and initial crack, as crack path is obtained as part of finite element approximation. The enrichment functions consist of the near-tip asymptotic function which capture the singularity and discontinuous function that describe the displacement jump across the crack surfaces. The approximation of displacement vector in partition of unity enrichment is [55],

$$u = \sum_{I=1}^{N} N_I(x)[u_I + H(x)a_I + \sum_{\alpha=1}^{4} F_\alpha(x)\, b_I^\alpha] \tag{2}$$

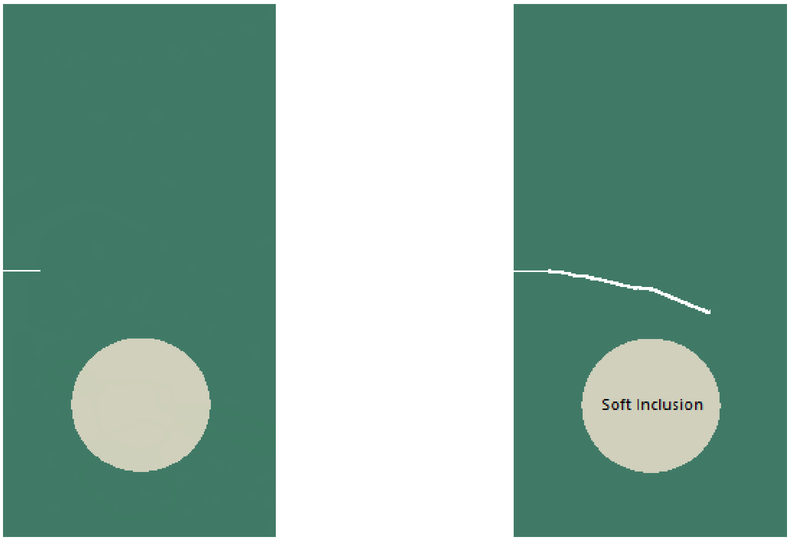
where $N_I(x)$ is the nodal shape functions; u_I is the nodal displacement vector; the second term is the product of the nodal enrichment degree of freedom vector, aI and the corresponding discontinuous jump function $H(x)$ across the crack surfaces; and the third term is the product of the nodal enrichment degree of freedom vector, b_I^α, and the corresponding elastic asymptotic crack-tip functions, $F_\alpha(x)$.

The extended finite element method (XFEM) in ABAQUS 6.14 is used in this simulation to study crack propagation in the presence of soft and hard inclusions. Two-dimensional rectangular specimen subjected to uniaxial tension is considered in this study. The effects of soft and hard inclusions on crack trajectory are investigated.

DISCUSSION

The results of all numerical simulations from soft and hard inclusions can be seen in Fig. 2 and 3. Figure 2 shows the crack propagation characteristic on the specimen when the soft inclusion is embedded in the materials. In the case of soft inclusion, the crack trajectory is tended to approach into the inclusion as soon as the crack propagates from an edge crack.

Whereas in the case of hard inclusion, the crack trajectory is tended to deflect away from the inclusion as soon as the crack propagates from an edge crack. The hard inclusion made the propagation path moved away from the inclusion's location as illustrated in Fig.3. These simulation results may be very significant in engineering materials design especially in predicting crack propagation in structural elements. These phenomena are in good agreement with the results presented by Bordas [24]. Both of results have a significantly similar in the crack propagation characteristics.



(a) (b)

Figure 2. Crack propagation interaction with soft inclusion. (a). Initial condition. (b) Final condition

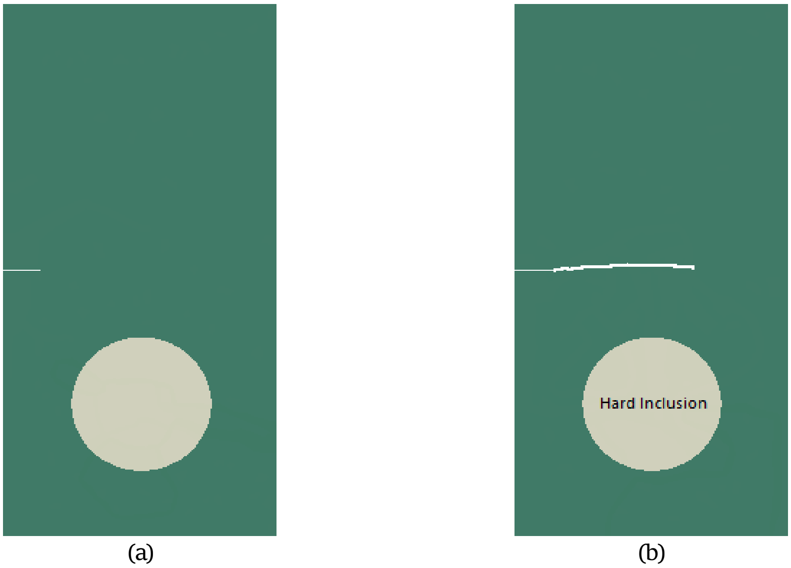


Figure 3. Crack propagation interaction with hard inclusion. (a). Initial condition. (b). Final condition

CONCLUSION

In this paper, the effect of presence of inclusion on the crack propagation is investigated numerically. The extended finite element method (XFEM) based on traction-separation cohesive behavior in Abaqus 6.14 is used in this study. Rectangular plate in two-dimension with single inclusion embedded off-centered is considered in this simulation. The specimen is pre-cracked with an edge crack and loaded with uniaxial tension. Two different cases, which are soft and hard inclusions, are considered based on different ratio of Young's modulus between the plate and the inclusion. The effect of presence of soft and hard inclusions on crack trajectory is investigated. Some conclusions can be made as follows:

1) The mismatch in Young's modulus between the plate and the embedded inclusion has significant effect on the crack trajectory of the specimen.
2) The specimen with embedded soft inclusion tends to attract the crack trajectory into the inclusion as soon as the crack initiates to propagate from an edge crack.
3) The specimen with embedded hard inclusion tends to deflect the crack trajectory away from the inclusion as soon as the crack initiates to propagate from an edge crack.

The present work focused on numerical modeling of crack growth in the presence of soft and hard inclusions using the extended finite element method (XFEM) in Abaqus 6.14. The advantage of using XFEM in the case of discontinuity problem compared to the conventional finite element techniques has shown in this paper. The future work will focus on investigating the effect of fracture strength from the inclusion on crack propagation of the specimen.

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